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Experimental design to persuade ☆

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ABSTRACT

A sender chooses ex ante how information will be disclosed ex post. A receiver obtains public information and information disclosed by the sender. Then he takes one of two actions. The sender wishes to maximize the probability that the receiver takes the desired action. The sender optimally discloses only whether the receiver's utility is above a cutoff. I derive necessary and sufficient conditions for the sender's and receiver's welfare to be monotonic in information. In particular, the sender's welfare increases with the precision of the sender's information and decreases with the precision of public information.

1. Introduction

Economists have long been interested in how an interested party can communicate her private information to a decision maker when their interests are imperfectly aligned (seminal contributions include Spence, 1973, Milgrom, 1981, and Crawford and Sobel, 1982). I study a situation in which before obtaining information, the interested party can choose a mechanism that specifies what information will be disclosed to the decision maker. This situation has been largely unexplored until recently (the pioneering articles are Rayo and Segal, 2010 and Kamenica and Gentzkow, 2011).

The drug approval process by the Food and Drug Administration (FDA) is a good example of such a situation. If a pharmaceutical company (manufacturer) wants a new drug to be approved, it has to submit a research protocol for all tests that are going to be undertaken. The research protocol includes not only tests chosen by the manufacturer but also standardized mandatory tests required by the FDA. The FDA closely monitors the record keeping and the adherence to the research protocol. So the FDA essentially observes both the design and results of all tests. Finally, based on the results of these tests, the FDA either approves the drug or rejects it. Because of the large cost of the process and large benefits of approval, the manufacturer has strong incentives to optimally design tests to maximize the probability of the FDA's approval.[1] What is the optimal design of tests? What determines the success rate of drug trials? What determines the average quality of approved drugs?

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[1] The description of the drug approval process is taken from Lipsky and Sharp (2001).

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I give exhaustive answers to these important questions by considering the following sender–receiver game. The receiver has a binary action choice: to act or not to act. The sender's utility depends only on the action taken by the receiver, and she prefers the receiver to act. The receiver's utility depends both on his action and on information. The receiver takes an action that maximizes his expected utility given his beliefs. He forms his beliefs based on public information and information disclosed by the sender. The sender chooses ex ante how information will be disclosed to the receiver ex post. Formally, she can publicly choose any conditional distribution of messages given information. I call this distribution a *mechanism*. The sender chooses the mechanism that maximizes her expected utility – the ex ante probability that the receiver will act. No monetary transfers between the sender and receiver are allowed.

This model is a special case of Kamenica and Gentzkow (2011) who consider a general model with an arbitrary set of actions, and arbitrary utility functions for the sender and receiver. They derive some interesting properties of the optimal mechanism. To completely characterize the optimal mechanism, I impose more structure that still fits many real-life examples well. Specifically, the optimal mechanism recommends the receiver to act when the receiver's utility given the information is above a cutoff and recommends the receiver not to act otherwise, where the cutoff is chosen to make the receiver exactly indifferent between the two actions when he is recommended to act.

The main contribution of the paper, however, is general monotone comparative statics results that relate the sender's and receiver's expected utilities at an optimal mechanism to the probability distribution of information. Specifically, I provide necessary and sufficient conditions for the sender and receiver to prefer one distribution of information to another for all values of the receiver's opportunity cost of acting. I now present the main results of the paper using the drug approval process.

What factors affect the manufacturer's welfare (or equivalently the probability of the drug approval)? The manufacturer's welfare is higher if the manufacturer is able to design more informative tests (in the mean-preserving spread sense) and if better drugs enter the testing phase (in the first-order stochastic dominance sense). Interestingly, under the absence of public information, these two conditions are not only sufficient but also necessary if the manufacturer's welfare is required to be higher for all values of the FDA's opportunity cost of approving the drug. Under the presence of public information, the manufacturer's welfare is higher if (and under some additional conditions, also only if) public information is less precise and more positive about the drug's quality.

What factors affect the FDA's welfare (or equivalently the expected quality of approved drugs)? Surprisingly, the FDA's welfare remains the same if the manufacturer is able to design more informative tests. However, the FDA's welfare is higher if public information is more precise and more positive about the drug's quality. These two conditions are also necessary if the FDA's welfare is required to be higher for all values of the opportunity cost of approving the drug. Finally, the overall welfare of the manufacturer and FDA is increasing in the precision of potential information of the manufacturer but is not monotonic in the precision of public information.

Although the above monotone comparative statics results are intuitive, they do not hold in the large existing literature where the sender chooses what information to disclose when she already has her private information. In particular, they do not hold under cheap talk and verifiable communication (Green and Stokey, 2007; Ivanov, 2010). The difference is due to the sender's incentive compatibility constraint on information disclosure, which is absent in my model, because the sender chooses what information to reveal at the ex ante stage.

Public information in the model captures not only information that will literally become public, such as the results of mandatory tests, but also any verifiable private information of the manufacturer or the FDA that they have at the ex ante stage, such as the results of preclinical trials or rival applications previously submitted to the FDA. Indeed, using an argument related to the unraveling argument of Milgrom (1981), I show that such information gets fully disclosed.

1.1. Related literature

The paper is related to two strands of the literature. The first strand of the literature studies optimal information disclosure games, in which the sender can commit to an information disclosure mechanism. The most influential paper in this strand is Kamenica and Gentzkow (2011) discussed above. Rayo and Segal (2010) and Kolotilin (2014) allow the receiver to have unverifiable private information. Gentzkow and Kamenica (2012) allow many senders to disclose information. Alonso and Câmara (2014a, 2014b) allow heterogeneous priors and heterogeneous receivers, respectively. Gentzkow and Kamenica (2014a) allow mechanisms to have different costs to the sender. Bergemann and Pesendorfer (2007) characterize optimal information disclosure in certain environments in which monetary transfers are allowed.

Lerner and Tirole (2006), Brocas and Carrillo (2007), and Benoit and Dubra (2011) study information disclosure in environments similar to mine, but in their models, the sender is exogenously constrained in choosing a mechanism; so they characterize a constrained, rather than unconstrained, optimal mechanism. Gill and Sgroi (2008), Gill and Sgroi (2012), Perez-Richet and Prady (2012), and Perez-Richet (2014) characterize constrained optimal mechanisms chosen by a privately informed sender. Glazer and Rubinstein (2004) characterize constrained optimal mechanisms chosen by the receiver rather than sender.

The second strand of the literature compares information structures in games. An information structure is associated with a distribution of types given the state. A seminal work of Blackwell (1953) shows that an information structure gives a higher optimal payoff than another one for all one-player games if and only if the latter is a garbling of the former.

Lehmann (1988) and Athey and Levin (2001) extend Blackwell (1953) by comparing information structures not for all one-player games but only for a class of monotone one-player games.

Hirshleifer (1971) notes that Blackwell (1953)'s result does not hold in many-player games: more information can make all players worse off in equilibrium. More information does not always increase the set of equilibrium outcomes because more information does not only increase the set of feasible outcomes but also adds incentive constraints. Bergemann and Morris (2014) abstract from feasibility issues by considering the weakest concept of correlated equilibrium and show that the set of equilibrium outcomes decreases in all games if and only if the information structure becomes more informative. Using Bayes Nash Equilibrium concept, in which both feasibility and incentive issues are present, Gossner (2000) characterizes when an information structure supports more equilibrium outcomes than another one for all games. Lehrer et al. (2013) characterize when two information structures support the same equilibrium outcomes for all games; they use many equilibrium concepts, including Bayes Nash Equilibrium and various notions of correlated equilibria surveyed in Forges (1993).

Considering a smaller class of games gives a sharper comparisons of information structures: Gossner and Mertens (2001) and Pęski (2008) consider zero-sum games and use a Bayes Nash Equilibrium concept; Lehrer et al. (2010) consider games with common interests and use many equilibrium concepts. These papers completely characterize the comparisons of information structures; the results are stated using various types of garblings, which are identical in case of one-player games.

In this paper, I consider another class of games: optimal information disclosure games with type-independent preferences of the sender and binary actions of the receiver. In contrast to the discussed papers, my class of games admits comparisons of distributions of types that may arise from different prior distributions of the states; that is, distributions of types can be compared in terms of not only informativeness but also optimism. I completely characterize when a sender/receiver prefers one distribution of types to another; the results are stated using various stochastic orders based on mean-preserving spread and first-order stochastic dominance. However, if compared distributions arise from the common prior distribution of the states, the results can be stated using garblings; the body of the paper discusses the relation between these results and the existing literature.

The rest of the paper is organized as follows. Section 2 presents the model and characterizes the optimal information disclosure mechanism. Section 3 presents the main results of the paper: monotone comparative statics. Section 4 concludes. All proofs, technical details, and supplementary results are relegated to the appendices.

2. Model

2.1. Setup

Consider a communication game between a female sender and a male receiver. The receiver takes a binary action $a = 0, 1$. Say that the receiver *acts* if he takes $a = 1$, and the receiver *does not act* if he takes $a = 0$. The sender's utility depends only on a, but the receiver's utility depends both on a and on (s, r), where components s and r denote the sender's type and public type, respectively. Without loss of generality, the sender's utility is a, and the receiver's utility is s if $a = 1$ and u_0 if $a = 0$.[2] Before (s, r) is realized, the sender can commit to a mechanism that sends a message m to the receiver as a (stochastic) function of (s, r); specifically, the sender chooses the conditional distribution $\phi(m|s, r)$ of m given (s, r).[3]

Assume that the set of messages M contains at least two elements m_0 and m_1, the set of sender's types S is $[\underline{s}, \bar{s}]$, the value of u_0 belongs to the set S, the set of public types R is an arbitrary set that satisfies mild regularity conditions that ensure that all conditional expectations exist.[4] The information (s, r) has some joint distribution. For simplicity, assume that all distributions admit strictly positive densities unless stated otherwise. In particular, the marginal distribution $G(r)$ of r and the conditional distribution $F(s|r)$ of s given r admit strictly positive densities $g(r)$ and $f(s|r)$.

The timing of the communication game is as follows:

1. The sender publicly chooses a mechanism $\phi(m|s, r)$.
2. A triple (m, s, r) is drawn according to ϕ, F, and G.
3. The receiver observes (m, r) and takes an action a.
4. Utilities of the sender and receiver are realized.

The solution concept used is Perfect Bayesian Equilibrium (PBE). I view PBEs as identical if they have the same equilibrium mapping from information (s, r) to the receiver's action a. At the third stage, the receiver forms a belief and acts if and only if the conditional expectation $\mathbb{E}_\phi[s|m, r]$ of s given (m, r) is at least u_0. (Note that PBE requires that the receiver takes the sender's preferred action whenever he is indifferent between the two actions.) At the first stage, the sender chooses an *optimal mechanism* that maximizes her expected utility, the probability that the receiver acts.

[2] In Online Appendix E, I allow the sender's utility to linearly depend on s and show how my methodology based on stochastic orders and some of the monotone comparative statics results generalize.

[3] The literature also refers to a mechanism as a disclosure policy, an information structure, a signal, and a test.

[4] For example, R is allowed to be a complete separable metric space endowed with the Borel sigma algebra (Theorems 1.4.12 and 4.1.6 in Durrett, 1996).

Using a similar argument to the revelation principle, restrict attention to direct mechanisms that send only two messages: m_0 that persuades the receiver not to act and m_1 that persuades the receiver to act. Adopt the convention that $\phi(m_1|s,r)$ denotes the probability of the message m_1 given (s,r). Hereafter, all notions are in the weak sense. For example, increasing means not decreasing and higher means not lower.

To see that my model is a good approximation of the drug approval process, let us reinterpret the manufacturer as the sender and the FDA as the receiver. The FDA's approval decision is the receiver's action, and the research protocol is the sender's choice of a mechanism. Any information that can potentially be revealed by some tests is the sender's information, the results of the mandatory tests is public information, and the results of the remaining tests is a message. The manufacturer has a lot of freedom in choosing the design of tests. For example, it chooses dosage and characteristics of volunteer patients, such as gender, age, and health condition (Lipsky and Sharp, 2001). Moreover, the manufacturer can make specifics of subsequent tests to be contingent on the results of the mandatory tests. Due to the FDA's regulation and close monitoring, the FDA observes both the design and results of all tests, and then approves the drug if its benefits outweigh its costs and risks.[5]

2.2. Optimal mechanism

The optimal mechanism ϕ^* has a simple cutoff structure.

Lemma 1. *The optimal mechanism is given by*

$$\phi^*(m_1|s,r) = \begin{cases} 1 & \text{if } s \geq s^*(r), \\ 0 & \text{if } s < s^*(r). \end{cases} \tag{1}$$

If $\mathbb{E}_F[s|r] \geq u_0$, *then* $s^*(r) = \underline{s}$; *otherwise* $s^*(r) < u_0$ *is the unique solution to equation* $\mathbb{E}_F[s - u_0|r, s \geq s^*(r)] = 0$.

Clearly, the optimal mechanism is conditioned on each piece of public information r. This implies that it does not matter whether the sender commits to a mechanism before or after the realization of r. I give the intuition for Lemma 1 conditional on some value r. If it is not possible to induce the receiver to always act, then the optimal mechanism induces the receiver to act if and only if his utility is above the cutoff. The cutoff is such that the receiver is indifferent between the two actions whenever he acts. Intuitively, the optimal mechanism has two defining properties. First, it makes the receiver indifferent between the two actions whenever he acts. Second, it makes the receiver know whether his utility is above the cutoff. If the first property were violated, then the receiver would strongly prefer to act whenever he acts. Thus, it would be possible to increase the probability that the receiver acts by sending m_1 for a slightly larger set of types s. If the second property were violated, then it would be possible to construct a mechanism that sends m_1 with the same total probability, but for higher types s. This mechanism would violate the first property, so it would be possible to increase the probability that the receiver acts.

Lemma 1 and subsequent results extend when the distribution of (s,r) does not admit a density, as I show in Online Appendix C. The only difference is that the optimal mechanism may randomize over messages at the cutoff as the following example shows. Suppose that $u_0 = 0$, public information is absent, and F is a discrete distribution that assigns probabilities $1/3$ and $2/3$ to 1 and -1. The optimal mechanism sends the message m_1 if $s = 1$, and the messages m_1 and m_0 with equal probabilities if $s = -1$. As a result, the receiver who gets m_1 is indifferent between the two actions and the probability of m_1 is $2/3$.

Versions of Lemma 1 appear in the literature, but my proof is simpler. Anderson and Renault (2006) establish that a cutoff mechanism is optimal in a similar setting but with transferable utilities and search costs. Lerner and Tirole (2006) show that the mechanism from Lemma 1 is optimal in a smaller class of feasible mechanisms in a more specific setting than mine. Benoit et al. (2008) and Kamenica and Gentzkow (2011) establish Lemma 1 for the above binary-type example. For a more general setting than mine, Kamenica and Gentzkow (2011) derive interesting properties of the optimal mechanism. In particular, these properties imply that m_1 makes the receiver indifferent between the two actions and that m_0 can only be sent from types $s < u_0$. However, they do not imply that the optimal mechanism has a cutoff structure in that m_0 is sent if and only if $s < s^*(r)$.

3. Comparative statics analysis

Because the optimal mechanism has a simple cutoff structure, the model is suitable for sharp comparative statics analysis. I first illustrate the main comparative statics results under the absence of public information. It is not trivial to generalize

[5] The drug approval process is used to illustrate general relevance of the model and provide interpretation of the results. A practitioner should note, however, that some of my results may be inconsistent with real-world stylized facts about the FDA because my model abstracts away from some aspects inherent in the FDA's drug approval process. For example, in reality, the FDA has some commitment power in its approval decision, the manufacturer cares not only about the probability of approval, and not all designs of tests are feasible. See Kamenica and Gentzkow (2011) for other real-world examples that fit the model.

the results when the public information is present because each public type generates a different distribution of the sender's type.[6] At the end of the section, I present this generalization and discuss its practical importance using the drug approval process.

3.1. Comparative statics without public information

In this section, assume the absence of public information. Proposition 1 presents monotone comparative statics results that relate the sender's and receiver's expected utilities under the optimal mechanism to the distribution of the sender's type. This proposition uses the standard definitions from the literature on stochastic orders. Let P_1 and P_2 be two distributions. *P_2 is higher than P_1 in the increasing convex order* if there exists a distribution P such that P_2 first-order stochastically dominates P and P is a mean-preserving spread of P_1.[7]

Proposition 1. *Let F_1 and F_2 be two distributions of s that do not depend on r.*

1. *The sender's expected utility under the optimal mechanism is higher under F_2 than under F_1 for all u_0 if and only if F_2 is higher than F_1 in the increasing convex order.*
2. *The receiver's expected utility under the optimal mechanism is higher under F_2 than under F_1 for all u_0 if and only if $\mathbb{E}_{F_2}[s] \geq \mathbb{E}_{F_1}[s]$.*

Part 1 states that the sender's expected utility is higher if the distribution of s is (i) more favorable for acting (in the first-order stochastic dominance sense) and (ii) more variable (in the mean-preserving spread sense). Condition (i) is straightforward: more favorable distribution makes it easier for the sender to persuade the receiver. As to condition (ii), shifting probability weights to the ends of the support of $[\underline{s}, \bar{s}]$ decreases $\mathbb{E}\left[s|s < F^{-1}(p^*)\right]$ and increases $\mathbb{E}\left[s|s \geq F^{-1}(p^*)\right]$ allowing the sender to increase the probability $1 - p^*$ that the receiver acts, as follows from Lemma 1. Interestingly, these two conditions are not only sufficient but also necessary if the sender's expected utility is required to be higher for any value of u_0.[8]

Part 2 holds because the optimal mechanism is as uninformative as possible from the receiver's perspective, as follows from Lemma 1. Indeed, under the optimal mechanism, if the receiver acts, then he either holds the prior belief or is indifferent between the two actions. Thus, the receiver's expected utility under the optimal mechanism is $\max\{\mathbb{E}[s], u_0\}$, which is equal to his expected utility under a mechanism that sends the same message regardless of s.

To get a deeper understanding of the results involving condition (ii) above, notice that the model has the following equivalent interpretation. There is an underlying binary state ω. The receiver's utility is ω if he acts (and u_0 if he does not). The sender's type s is a noisy signal about ω with normalization $s = \mathbb{E}[\omega|s]$. The sender chooses a mechanism $\phi(m|s)$, which determines how much information about s is disclosed to the receiver. Indeed, let ω take values $\underline{\omega} = \underline{s}$ and $\overline{\omega} = \bar{s}$ with probabilities $\underline{p} = \left(\bar{s} - \mathbb{E}[s]\right) / \left(\bar{s} - \underline{s}\right)$ and $\overline{p} = 1 - \underline{p}$; let the density functions of s given ω be $h\left(s|\underline{\omega}\right) = f(s)\left(\bar{s} - s\right) / \left(\bar{s} - \mathbb{E}[s]\right)$ and $h(s|\overline{\omega}) = f(s)\left(s - \underline{s}\right) / \left(\mathbb{E}[s] - \underline{s}\right)$. For this construction, $\underline{p}h\left(s|\underline{\omega}\right) + \overline{p}h(s|\overline{\omega}) = f(s)$ and $\mathbb{E}[\omega|s] = s$, which establishes the equivalence.

Under this interpretation, variability condition (ii) corresponds to an informativeness of potential information s (Blackwell, 1953). The following corollary of Proposition 1 presents comparative statics with respect to such changes in informativeness.

Corollary 1. *Let the receiver's utility from acting ω be either $\underline{\omega}$ or $\overline{\omega}$ with probabilities $\underline{p}$ and $\overline{p}$. Let $H_1(s|\omega)$ and $H_2(s|\omega)$ be two conditional distributions of s given ω that admit densities $h_1(s|\omega)$ and $h_2(s|\omega)$.*

1. *The sender's expected utility under the optimal mechanism is higher under H_2 than under H_1 for all u_0 if and only if there exists a distribution $Q(s_1|s_2)$ of s_1 given s_2 such that for all ω and s_1,*

$$H_1(s_1|\omega) = \int Q(s_1|s_2)\, h_2(s_2|\omega)\, ds_2. \tag{2}$$

2. *The receiver's expected utility under the optimal mechanism is the same under H_2 and under H_1 for all u_0.*

In part 1, the distribution H_1 is a garbling of the distribution H_2 in that H_1 is obtained from H_2 by adding noise. Thus, any mechanism $\phi_1(m|s_1)$ under H_1 can be replicated under H_2 by $\phi_2(m|s_2) = \int \phi_1(m|s_1)\, dQ(s_1|s_2)$, which implies that the sender's expected utility is higher under a more informative distribution H_2. Notice that this replication argument does

[6] Therefore, to compare two distributions of information one needs to compare two distributions of distributions of the sender's type instead of simply comparing two distributions of the sender's type as under the absence of public information.

[7] See Definition 1 and Theorem 1 in Appendix A for more definitions and results on stochastic orders.

[8] As can be seen from the proof, it is straightforward to write a strong version of Proposition 1 in which the sender's and receiver's expected utilities are strictly higher under F_2.

not rely on the specific assumptions imposed on the class of information disclosure games that I consider. Therefore, the sender's expected utility increases with the precision of her information even if action a is not binary and the sender's and receiver's utilities depend on a and s in an arbitrary way.[9]

Part 2 holds again because the optimal mechanism leaves no rent to the receiver, so the receiver's expected utility $\max\{\mathbb{E}[\omega], u_0\}$ does not depend on the distribution of s. However, if the sender's utility depended on s, the optimal mechanism could leave some rent to the receiver; so the receiver's expected utility would change ambiguously with the precision of the sender's information (see Online Appendix E).

The monotone comparative statics results, however, crucially rely on the assumption that the sender can choose any mechanism at the ex ante stage. Under a cheap talk version of my model, the sender would not be able to disclose any information because she always prefers the receiver to act. Thus, the sender's expected utility would not change as her information becomes more precise. More generally, Green and Stokey (2007) and Ivanov (2010) show that the sender's ex ante expected utility may strictly decrease in the precision of her information in the case of unverifiable information. This happens because having less precise information may reduce the sender's incentive to misrepresent information.

Under a verifiable communication version of my model, there exists an equilibrium in which the sender discloses all her information. Following the literature, this paragraph focuses on this fully revealing equilibrium.[10] Thus, by Lemma 1, it is optimal for the sender to know only whether the receiver's utility is above the cutoff s^*, which is less informative than knowing the receiver's utility exactly. That is, the sender's ex ante expected utility may strictly decrease in the precision of her information in the case of verifiable information.

In the drug approval process, the manufacturer's welfare is the success rate of drug trials and the FDA's welfare is the average quality of approved drugs. Note that these variables can actually be observed in the data, so the theoretical comparative statics results are amenable to empirical analysis. Others being equal, the success rate is higher if the manufacturer has a better drug discovery process, or if the manufacturer has better testing capabilities. The average quality of approved drugs is also higher under the first scenario, but is the same under the second scenario.

3.2. Comparative statics with public information

This section generalizes the previous section by introducing public information. Notice that each piece of public information r is associated with a distinct conditional distribution $F(.|r)$ of the sender's type s. For convenience, I identify r with $F(.|r)$ so that the only primitive of the model is a distribution G of r. Thus, to compare two environments, we need to compare two distributions G of r, where r is multidimensional because it is a distribution of s given r.

To avoid technical issues that arise due to multidimensionality, I start with the case of binary s. Specifically, assume that $s = \underline{s}, \bar{s}$ and $r = \Pr(\bar{s}|r)$. Proposition 2 presents monotone comparative statics results that relate the sender's and receiver's expected utilities under the optimal mechanism to the primitive G. This proposition uses a new stochastic order. *P_2 is higher than P_1 in the increasing concave order* if there exists P such that P_2 first-order stochastically dominates P and P_1 is a mean-preserving spread of P.[11]

Proposition 2. *Let the support of $F(.|r)$ consist of $\underline{s}$ and $\bar{s}$ where r is identified with $\Pr(\bar{s}|r)$ for all $r \in R = [0, 1]$. Let G_1 and G_2 be two distributions of r.*

1. *The sender's expected utility under the optimal mechanism is higher under G_2 than under G_1 for all u_0 if and only if G_2 is higher than G_1 in the increasing concave order.*
2. *The receiver's expected utility under the optimal mechanism is higher under G_2 than under G_1 for all u_0 if and only if G_2 is higher than G_1 in the increasing convex order.*

Part 1 of Proposition 2 states that the sender's expected utility increases as the distribution of public information becomes (i) more favorable for acting (in the first-order stochastic dominance sense) and (ii) less variable (in the mean-preserving spread sense). The intuition for condition (i) is again straightforward: more favorable public information makes it easier for the sender to persuade the receiver. As to condition (ii), when public information is less polarized, the receiver has a weaker opinion about his best action, so it is easier for the sender to influence him.

Part 2 of Proposition 2 states that the receiver's expected utility increases as the distribution G of public information becomes (i) more favorable for acting and (ii) more variable. As before the receiver has the same expected utility under the optimal mechanism and the mechanism ϕ_0 that sends the same message regardless of s. Since the receiver's utility

[9] This result (sufficiency of (2) in Corollary 1 for part 1) is an implication of Blackwell (1953). However, Proposition 1 and necessity of (2) in Corollary 1 are new to the literature to the best of my knowledge.

[10] This equilibrium can be supported by maximally skeptical out-of-equilibrium beliefs of the receiver in that the receiver believes that the actual type of the sender is equal to the minimum type consistent with the sender's report. Milgrom and Roberts (1986) argue that skeptical beliefs are most plausible in many situations. Miura (2014) shows that the outcome of the fully revealing equilibrium in binary-action games is the unique one that survives a *certifiable dominance* refinement. Moreover, most of the literature on verifiable communication focuses on the fully revealing equilibrium even though other equilibria may exist (see, for example, Okuno-Fujiwara et al., 1990, Seidmann and Winter, 1997, and Hagenbach et al., 2014).

[11] The increasing concave order is also known as the second-order stochastic dominance.

from not acting is fixed at u_0, the receiver is better off as G becomes more favorable for acting by the revealed preference argument. As to condition (ii), the receiver with a stronger opinion enjoys a higher expected utility from his preferred action. Interestingly, conditions (i) and (ii) for both parts of Proposition 2 are not only sufficient but also necessary if the sender and receiver are required to be better off under G_2 for all values of u_0.

Mathematically, Proposition 2 is exhaustive because it gives tight comparative statics results with respect to the primitive G. But economically, changes in G in the first-order stochastic dominance sense are not meaningful if the prior distribution of s is fixed. Indeed, if the prior probability of $\bar{s}$ is fixed at $\bar{p}$ and r is a public signal about s (normalized as before to $\Pr(\bar{s}|r)$), then any feasible distribution G of r must satisfy $\int r dG(r) = \bar{p}$. Therefore, changes in G should be mean-preserving under this interpretation of public information.

Mean-preserving changes in G correspond to variability condition (ii), which in turn corresponds to informativeness of public information (Blackwell, 1953). The following corollary of Proposition 2 presents comparative statics with respect to changes in informativeness.

Corollary 2. *Let s take only two values $\underline{s}$ and $\bar{s}$ where $\Pr(\bar{s}) = \bar{p}$. Let H_1 and H_2 be two distributions of public signals r_1 and r_2 given s. The sender's (receiver's) expected utility under the optimal mechanism is higher (lower) under H_2 than under H_1 for all u_0 if and only if there exists a distribution $Q(r_2|r_1)$ of r_2 given r_1 such that for all s and r_2,*

$$H_2(r_2|s) = \int Q(r_2|r_1) h_1(r_1|s) dr_1. \tag{3}$$

The corollary states that the sender is worse off and the receiver is better off when public information is more precise.[12] Indeed, public information is less precise under H_2 because H_2 only adds noise to H_1. Intuitively, under H_2, the sender can replicate any mechanism ϕ_1 available under H_1 by using a two stage mechanism. The first stage of this mechanism will make public information more precise, from H_2 to H_1, and the second stage will implement ϕ_1 given the more precise public information H_1. The receiver, in contrast, prefers more precise public information H_1 because he can take a better informed action under H_1 and the optimal mechanism leaves no rent to him.

Proposition 3 shows that in the general case of continuous s, changes in the distribution G according to conditions (i) and (ii) have the same qualitative effects on the sender's and receiver's expected utilities as in Proposition 2. However, in this case, r being a distribution of s becomes infinite dimensional, so we need to extend the stochastic orders to the multidimensional case and impose a partial order on R.[13]

Proposition 3. *Let R be the set of distributions on $[\underline{s}, \bar{s}]$ endowed with an increasing convex order in that r_2 is higher than r_1 if $F(.|r_2)$ is higher than $F(.|r_1)$ in the increasing convex order. Let G_1 and G_2 be two distributions of r.*

1. *The sender's expected utility under the optimal mechanism is higher under G_2 than under G_1 for all u_0 if G_2 is higher than G_1 in the increasing concave order.*
2. *The receiver's expected utility under the optimal mechanism is higher under G_2 than under G_1 for all u_0 if G_2 is higher than G_1 in the increasing convex order.*

An important implication of Proposition 3 is that the sender becomes worse off and the receiver better off as public information becomes more precise.

Corollary 3. *Let the prior density of s be given by f. Let h_1 and h_2 be two densities of public signals r_1 and r_2 given s. The sender's (receiver's) expected utility under the optimal mechanism is higher (lower) under h_2 than under h_1 for all u_0 if there exists a density $q(r_2|r_1)$ of r_2 given r_1 such that for all s and r_2,*

$$h_2(r_2|s) = \int q(r_2|r_1) h_1(r_1|s) dr_1. \tag{4}$$

One key step of the proof shows that the sender's expected utility is concave in the prior F. This proof does not rely on the specific assumptions imposed on the class of information disclosure games that I consider. Therefore, the sender's expected utility decreases with the precision of public information even if action a is not binary and the sender's and receiver's utilities depend on a and s in an arbitrary way.[14]

Another key step of the proof shows that the receiver's expected utility is convex in the prior F. This result continues to hold as long as action a is binary (see Online Appendix E). Therefore, the receiver's expected utility increases with the

[12] Again, in the cheap talk literature, the sender's and receiver's expected utilities are not monotonic in the precision of public information (Chen, 2012).

[13] See Appendix A for definitions and results on multidimensional stochastic orders and Online Appendix D for discussion of Proposition 3.

[14] This result (the sender's part of Corollary 3) can also be obtained using the independent work of Bergemann and Morris (2014), who show that the set of equilibrium outcomes decreases in all games if and only if the information structure becomes more informative. However, Propositions 2, 3, the receiver's parts of Corollaries 2, 3, and necessity of (3) in Corollary 2 are new to the literature to the best of my knowledge.

precision of public information even if the sender's and receiver's utilities depend on a and s in an arbitrary way provided that action a is binary.

The social welfare, however, does not necessarily increase with the precision of public information even if it puts a very small weight on the sender's utility. Indeed, suppose that initially public information is absent. As public information appears, the marginal increase in the receiver's expected utility is 0 by the Envelope Theorem, as noted by Radner and Stiglitz (1984), but the marginal decrease in the sender's expected utility is strictly positive.

Continuing the drug approval process example, any commonly known information at the time the manufacturer designs drug trials or any information that the FDA requires to be revealed during drug trials can be viewed as public information. The above results mean that requiring the manufacturer to run more tests, increases the average quality of approved drugs but decreases the success rate of drug trials. Moreover, any verifiable private information of the manufacturer or FDA that they have at the ex ante stage is fully disclosed in the unique equilibrium, as I show in Online Appendices F and G.[15] Therefore, any such information can be viewed as public information and all results from this section continue to hold.[16] In particular, Corollary 3 then implies that the average quality of approved drugs increases and the success rate decreases if the manufacturer carries out more thorough preclinical trials or if the FDA has more precise private information about the tested drug from other sources, such as from rival applications previously submitted to the FDA.

4. Conclusions

In this paper, I have studied optimal information disclosure mechanisms. I have imposed the following key assumptions. First, at the ex ante stage, the sender can publicly choose how her information will be disclosed ex post; specifically, she can choose any conditional distribution of messages given her information and exogenous public information. Second, the receiver has a binary action choice. Third, the sender's utility depends on the receiver's action but does not depend on information.

The model is highly tractable and can be used as a building block. Compared to no disclosure, the optimal disclosure mechanism gives the same expected utility to the receiver but persuades him to act with a higher probability. Thus, in a richer model with many receivers, the receivers will gain or lose from the optimal information disclosure mechanism depending only on whether externalities from acting are positive or negative.

The monotone comparative statics results imply that it is straightforward to enrich the model with strategic decisions of the sender and receiver that affect the information structure. Returning to the drug approval process example, the manufacturer can increase the success rate of drug trials by improving the testing phase such that a better design of drug trials can be chosen. This improvement, however, does not affect the quality of approved drugs. The FDA can improve the quality of approved drugs by imposing more mandatory tests that the manufacturer must carry out, thereby obtaining more precise public information about the tested drug. Imposing more mandatory tests, however, decreases the success rate of drug trials and may discourage the manufacturer from going through the costly drug approval process, which may in turn hurt the social welfare.

I have also shown that the sender and receiver disclose all of their verifiable private information at the ex ante stage, so all of my results apply after reinterpreting this private information as public. However, in this paper, I have not explored the possibility of the receiver having private information that cannot be elicited by the sender ex ante. Generically, the receiver does have such private information at least by the time he takes an action. For example, the FDA carries out an independent review after receiving the application from the manufacturer. Moreover, the manufacturer is uncertain about preferences and beliefs of the FDA regarding the safety and efficacy of a new drug. Since the optimal mechanism leaves no rent to the receiver if the receiver is uninformed, as a trivial result, the optimal mechanism is (weakly) more informative if the receiver is privately informed. The detailed analysis of this situation is my central goal in Kolotilin (2014).

Appendix A. Stochastic orders

Definition 1 presents the unidimensional stochastic orders used in Section 3.1.

Definition 1. Let X_1 and X_2 be two random variables with distributions P_1 and P_2 on $[\underline{x}, \bar{x}]$. Say that

[15] These results resemble the unraveling result of Milgrom (1981) who study verifiable communication games. The proofs, however, are unlike the proof of Milgrom (1981), because in my model the order of play is different and the sender has a commitment power. The case of the receiver's verifiable information studied in Online Appendix F is related to the literature on mechanism design with evidence (Green and Laffont, 1986; Bull and Watson, 2007; Deneckere and Severinov, 2008; Kartik and Tercieux, 2012; Koessler and Perez-Richet, 2014), where a principal commits to a mechanism that maps agents' verifiable reports to principal's actions. The case of the sender's verifiable information studied in Online Appendix G is related to the literature on mechanism design by an informed principal (Myerson, 1983; Maskin and Tirole, 1990, 1992) with the difference that the principal's information is unverifiable in this literature.

[16] Relatedly, the reader may wonder whether the main results of the paper would change if the message m generated by the mechanism ϕ was privately observed by the sender. Gentzkow and Kamenica (2014b) show that the optimal mechanism does not change and m is fully disclosed by the sender if m is verifiable.

1. P_2 first-order stochastically dominates P_1 (denoted by $P_2 \geq_{st} P_1$) if $P_2(x) \leq P_1(x)$ for all x.
2. P_2 is a mean-preserving spread of P_1 (denoted by $P_2 \geq_{cx} P_1$) if there exist two random variables $\widehat{X}_2$ and $\widehat{X}_1$, defined on the same probability space, with distributions P_2 and P_1 such that $\mathbb{E}\left[\widehat{X}_2|\widehat{X}_1\right] = \widehat{X}_1$.
3. P_2 is higher than P_1 in the increasing convex order (denoted by $P_2 \geq_{icx} P_1$) if there exists a distribution P such that $P_2 \geq_{st} P \geq_{cx} P_1$.
4. P_2 is higher than P_1 in the increasing concave order (denoted by $P_2 \geq_{icv} P_1$) if there exists a distribution P such that $P_2 \geq_{st} P$ and $P_1 \geq_{cx} P$.

Theorem 1 gives useful equivalent representations of the above stochastic orders.

Theorem 1. *Let P_1 and P_2 be two distributions that admit densities on $[\underline{x}, \bar{x}]$.*

1. $P_2 \geq_{st} P_1$ *if and only if* $\mathbb{E}[h(X_2)] \geq \mathbb{E}[h(X_1)]$ *for all increasing functions* $h : [\underline{x}, \bar{x}] \to \mathbb{R}$.
2. $P_2 \geq_{cx} P_1$ *is equivalent to each of the following conditions:*
   (a) $\mathbb{E}[h(X_2)] \geq \mathbb{E}[h(X_1)]$ *for all convex functions* $h : [\underline{x}, \bar{x}] \to \mathbb{R}$;
   (b) $\int_x^{\bar{x}} P_2(\widetilde{x})\, d\widetilde{x} \leq \int_x^{\bar{x}} P_1(\widetilde{x})\, d\widetilde{x}$ *for all* $x \in [\underline{x}, \bar{x}]$ *with equality for* $x = \underline{x}$;
   (c) $\int_p^1 P_2^{-1}(\widetilde{p})\, d\widetilde{p} \geq \int_p^1 P_1^{-1}(\widetilde{p})\, d\widetilde{p}$ *for all* $p \in [0, 1]$ *with equality for* $p = 0$.
3. $P_2 \geq_{icx} P_1$ *is equivalent to each of the following conditions:*
   (a) $\mathbb{E}[h(X_2)] \geq \mathbb{E}[h(X_1)]$ *for all increasing convex functions* $h : [\underline{x}, \bar{x}] \to \mathbb{R}$;
   (b) $\int_x^{\bar{x}} P_2(\widetilde{x})\, d\widetilde{x} \leq \int_x^{\bar{x}} P_1(\widetilde{x})\, d\widetilde{x}$ *for all* $x \in [\underline{x}, \bar{x}]$;
   (c) $\int_p^1 P_2^{-1}(\widetilde{p})\, d\widetilde{p} \geq \int_p^1 P_1^{-1}(\widetilde{p})\, d\widetilde{p}$ *for all* $p \in [0, 1]$.
4. $P_2 \geq_{icv} P_1$ *is equivalent to each of the following conditions:*
   (a) $\mathbb{E}[h(X_2)] \geq \mathbb{E}[h(X_1)]$ *for all increasing concave functions* $h : [\underline{x}, \bar{x}] \to \mathbb{R}$;
   (b) $\int_{\underline{x}}^{x} P_2(\widetilde{x})\, d\widetilde{x} \leq \int_{\underline{x}}^{x} P_1(\widetilde{x})\, d\widetilde{x}$ *for all* $x \in [\underline{x}, \bar{x}]$;
   (c) $\int_0^p P_2^{-1}(\widetilde{p})\, d\widetilde{p} \geq \int_0^p P_1^{-1}(\widetilde{p})\, d\widetilde{p}$ *for all* $p \in [0, 1]$.

Proof. See Shaked and Shanthikumar (2007) Section 1.A.1 for part 1, Section 3.A.1 for part 2, and Section 4.A.1 for parts 3 and 4. □

Definition 2 presents the multidimensional stochastic orders used in Section 3.2.

Definition 2. Let $\mathcal{P}$ be the set of distributions on $[\underline{x}, \bar{x}]$ endowed with some partial order $\geq_P$. Let $\mathbf{X}_1$ and $\mathbf{X}_2$ be two random elements with distributions Q_1 and Q_2 on $\mathcal{P}$. Say that

1. Q_2 first-order stochastically dominates Q_1 (denoted by $Q_2 \geq_{mst} Q_1$) if $\Pr_{Q_2}(\mathbf{X}_2 \in U) \geq \Pr_{Q_1}(\mathbf{X}_1 \in U)$ for all measurable increasing sets $U \subset \mathcal{P}$ in that $P \geq_P P'$ and $P' \in U$ imply $P \in U$.
2. Q_2 is a mean-preserving spread of Q_1 (denoted by $Q_2 \geq_{mcx} Q_1$) if there exist two random elements $\widehat{\mathbf{X}}_2$ and $\widehat{\mathbf{X}}_1$, defined on the same probability space, with distributions Q_2 and Q_1 such that $\mathbb{E}\left[\widehat{\mathbf{X}}_2|\widehat{\mathbf{X}}_1\right] = \widehat{\mathbf{X}}_1$.
3. Q_2 is higher than Q_1 in the increasing convex order (denoted by $Q_2 \geq_{micx} Q_1$) if there exists a distribution Q such that $Q_2 \geq_{mst} Q \geq_{mcx} Q_1$.
4. Q_2 is higher than Q_1 in the increasing concave order (denoted by $Q_2 \geq_{micv} Q_1$) if there exists a distribution Q such that $Q_2 \geq_{mst} Q$ and $Q_1 \geq_{mcx} Q$.

Theorem 2 gives equivalent representations of the multidimensional stochastic orders.

Theorem 2. *Let Q_1 and Q_2 be two distributions on $\mathcal{P}$.*

1. $Q_2 \geq_{mst} Q_1$ *if and only if* $\mathbb{E}[h(\mathbf{X}_2)] \geq \mathbb{E}[h(\mathbf{X}_1)]$ *for all increasing functions* $h : \mathcal{P} \to \mathbb{R}$ *in that* $h(P_2) \geq h(P_1)$ *for all* $P_1, P_2 \in \mathcal{P}$ *such that* $P_2 \geq_P P_1$.
2. $Q_2 \geq_{mcx} Q_1$ *if and only if* $\mathbb{E}[h(\mathbf{X}_2)] \geq \mathbb{E}[h(\mathbf{X}_1)]$ *for all convex functions* $h : \mathcal{P} \to \mathbb{R}$ *in that* $h(\alpha P_1 + (1-\alpha) P_2) \leq \alpha h(P_1) + (1-\alpha) h(P_2)$ *for all* $P_1, P_2 \in \mathcal{P}$ *and all* $\alpha \in (0, 1)$.
3. $Q_2 \geq_{micx} Q_1$ *if and only if* $\mathbb{E}[h(\mathbf{X}_2)] \geq \mathbb{E}[h(\mathbf{X}_1)]$ *for all increasing convex functions* $h : \mathcal{P} \to \mathbb{R}$.
4. $Q_2 \geq_{micv} Q_1$ *if and only if* $\mathbb{E}[h(\mathbf{X}_2)] \geq \mathbb{E}[h(\mathbf{X}_1)]$ *for all increasing concave functions* $h : \mathcal{P} \to \mathbb{R}$.

Proof. See Shaked and Shanthikumar (2007) Section 6.B.1 for part 1, and Section 7.A.1 for parts 2, 3, and 4. □

Appendix B. Proofs

Proof of Lemma 1. The optimal mechanism ϕ^* solves

$$\underset{\phi(m_1|s,r)\in[0,1]}{\text{maximize}} \int_{S\times R} f(s|r)\, g(r)\, \phi(m_1|s,r)\, drds$$

subject to

$$\int_S (s-u_0)\, f(s|r)\, \phi(m_1|s,r)\, ds \geq 0 \text{ for all } r\in R$$

where the objective function is the probability that the receiver acts and the constraint requires that the receiver prefers to act whenever he receives m_1.

The Lagrangian for this problem is given by:

$$\mathcal{L} = \int_{S\times R} (1+[s-u_0]\,\lambda(r))\, f(s|r)\, g(r)\, \phi(m_1|s,r)\, drds,$$

where $\lambda(r)\, g(r)$ is a multiplier for the constraint. Since the choice variable $\phi(m_1|s,r)$ belongs to the unit interval, we have $\phi(m_1|s,r)=1$ if $s\geq u_0-1/\lambda(r)$ and $\phi(m_1|s,r)=0$ if $s<u_0-1/\lambda(r)$ where $\lambda(r)$ is 0 if $\mathbb{E}_F[s|r]>u_0$ and is such that the constraint is binding if $\mathbb{E}_F[s|r]\leq u_0$. □

Proof of Proposition 1. I start by proving the first part. Let s_i^* be given by Lemma 1 where F is replaced with F_i. If $F_2 \geq_{icx} F_1$ (see Definition 1), then the sender can induce the receiver to act with a higher probability under F_2 than under F_1 because

$$\int_{F_2^{-1}(F_1(s_1^*))}^{\bar{s}} (s-u_0)\, dF_2(s) = \int_{F_1(s_1^*)}^{1} \left(F_2^{-1}(\tilde{p})-u_0\right) d\tilde{p} \geq \int_{F_1(s_1^*)}^{1} \left(F_1^{-1}(\tilde{p})-u_0\right) d\tilde{p} = \int_{s_1^*}^{\bar{s}} (s-u_0)\, dF_1(s) \geq 0,$$

where the equalities hold by the appropriate change of variables, the first inequality holds by Theorem 1 part 3 (c), and the last inequality holds by Lemma 1. Conversely, if $F_2 \ngeq_{icx} F_1$, then by Theorem 1 part 3 (c), there exists p such that $\int_p^1 F_2^{-1}(\tilde{p})\, d\tilde{p} < \int_p^1 F_1^{-1}(\tilde{p})\, d\tilde{p}$. Setting $u_0 = \int_{F_2^{-1}(p)}^1 s dF_2(s)/(1-p)$ and using an analogous argument, we get that the receiver acts with a strictly higher probability under F_1 than under F_2:

$$\int_{F_1^{-1}(p)}^{\bar{s}} (s-u_0)\, dF_1(s) = \int_p^1 \left(F_1^{-1}(\tilde{p})-u_0\right) d\tilde{p} > \int_p^1 \left(F_2^{-1}(\tilde{p})-u_0\right) d\tilde{p} = \int_{F_2^{-1}(p)}^{\bar{s}} (s-u_0)\, dF_2(s) = 0.$$

Now I prove the second part. The receiver's expected utility under F_i is $\max\left\{\mathbb{E}_{F_i}[s], u_0\right\}$ by Lemma 1. Clearly, if $\mathbb{E}_{F_2}[s]\geq \mathbb{E}_{F_1}[s]$, then $\max\left\{\mathbb{E}_{F_2}[s], u_0\right\} \geq \max\left\{\mathbb{E}_{F_1}[s], u_0\right\}$ for all u_0. Conversely, if $\mathbb{E}_{F_2}[s] < \mathbb{E}_{F_1}[s]$, then $\max\left\{\mathbb{E}_{F_2}[s], u_0\right\} < \max\left\{\mathbb{E}_{F_1}[s], u_0\right\}$ for any $u_0\in\left(\mathbb{E}_{F_2}[s], \mathbb{E}_{F_1}[s]\right)$. □

Proof of Corollary 1. The distribution of the posterior $\Pr(\overline{\omega}|s)$ under H_2 is a mean-preserving spread of that under H_1 if and only if there exists Q such that (2) holds, as Blackwell (1953) shows. Since the posterior $\Pr(\overline{\omega}|s) = \left(s-\underline{\omega}\right)/\left(\overline{\omega}-\underline{\omega}\right)$ is linear in s, the distribution of the posterior $\Pr(\overline{\omega}|s)$ under H_2 is a mean-preserving spread of that under H_1 if and only if $F_2 \geq_{cx} F_1$ where F_i is the distribution of s under H_i, given by $F_i(s) = H_i\left(s|\underline{\omega}\right)\underline{q} + H_i(s|\overline{\omega})\,\bar{q}$ for $i=1,2$. Part 1 then follows by repeating all steps of the proof of Proposition 1 with the only difference that Theorem 1 part 2 (c) is used instead of Theorem 1 part 3 (c). Part 2 holds because the receiver's expected utility is $\max\{\mathbb{E}[\omega], u_0\}$ by Lemma 1, and $\mathbb{E}[\omega] = \underline{\omega}\underline{q} + \overline{\omega}\bar{q}$ does not depend on H. □

Proof of Proposition 2. The sender's expected utility under the optimal mechanism is:

$$U_S = \int_R \min\left\{\frac{\bar{s}-\underline{s}}{u_0-\underline{s}}\Pr\left(\bar{s}|r\right), 1\right\} dG(r) = \int_0^1 \min\left\{\frac{\bar{s}-\underline{s}}{u_0-\underline{s}} r, 1\right\} dG(r) = 1 - \frac{\bar{s}-\underline{s}}{u_0-\underline{s}} \int_0^{\frac{u_0-\underline{s}}{\bar{s}-\underline{s}}} G(r)\, dr, \tag{5}$$

where the first equality holds by Lemma 2 from Online Appendix C, the second by convention $\Pr\left(\bar{s}|r\right)=r$, and the third by integration by parts. Part 1 of the proposition follows immediately by Theorem 1 part 4 (b).

The receiver's expected utility under the optimal mechanism is:

$$U_R = \int_R \max\{u_0, \mathbb{E}[s|r]\}\, dG(r) = \int_0^1 \max\{u_0, \underline{s} + (\bar{s} - \underline{s})\, r\}\, dG(r) = \bar{s} - (\bar{s} - \underline{s}) \int_{\frac{u_0 - \underline{s}}{\bar{s} - \underline{s}}}^{1} G(r)\, dr, \tag{6}$$

where the first equality holds by Lemma 2 from Online Appendix C, the second by convention $\Pr(\bar{s}|r) = r$, and the third by integration by parts. Part 2 of the proposition follows immediately by Theorem 1 part 3 (b). □

Proof of Corollary 2. The distribution of the posterior $\Pr(\bar{s}|r)$ under H_1 is a mean-preserving spread of that under H_2 if and only if there exists Q such that (3) holds, as Blackwell (1953) shows. Applying Lemma 1 part 2 (b) to (5) and (6) proves the corollary. □

Proof of Proposition 3. The sender's expected utility is $\int_R U_S^*(r)\, dG(r)$ where $U_S^*(r)$ is the sender's expected utility conditional on r. The function U_S^* is increasing in r in the increasing convex order by Proposition 1 part 1. Moreover, U_S^* is concave in r, because there exists a mechanism ϕ that gives the sender expected utility $\alpha U_S^*(r_1) + (1-\alpha)\, U_S^*(r_2)$ when the distribution of s is $\alpha F(s|r_1) + (1-\alpha)\, F(s|r_2)$. The required mechanism is simply a mechanism that implements ϕ_1^* and ϕ_2^* with probabilities α and $1-\alpha$. Therefore, part 1 of the proposition follows by Theorem 2 part 4.

The receiver's expected utility under the optimal mechanism is

$$U_R = \int_R \max\{u_0, \mathbb{E}[s|r]\}\, dG(r)\,.$$

The function $\mathbb{E}[s|r]$ is linear in r because the expectation is linear in $F(.|r)$, which is identified with r. Moreover, $\mathbb{E}[s|r]$ is increasing in r by Theorem 1 part 3 (a) because $h(s) = s$ is increasing and convex in s and R is endowed with an increasing convex order. Thus, the function $\max\{u_0, \mathbb{E}[s|r]\}$ is increasing and convex in r. Part 2 of the proposition follows by Theorem 2 part 3. □

Proof of Corollary 3. By Le Cam (1964), who extend Blackwell (1953) to the infinite state space case, (4) implies $G_1 \geq_{mcx} G_2$. Therefore, by Definition 2, $G_2 \geq_{micv} G_1$ and $G_1 \geq_{micx} G_2$; so applying Proposition 3 proves the corollary. □

Appendix. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.geb.2015.02.006.

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