Available online at www.sciencedirect.com
ScienceDirect
Games and Economic Behavior 00 (2015) 1–7
Games Econ. Behav.

Online appendices to "Experimental design to persuade"

Anton Kolotilin

University of New South Wales, School of Economics, Australia

Abstract

Online appendices generalize the analysis to discontinuous distributions, multidimensional types, type-dependent preferences, and verifiable private information of the sender and receiver.

Appendix C. Discontinuous distributions

This appendix relaxes the assumption that all distributions are continuous. Instead, assume that $G(r)$ and $F(s|r)$ are arbitrary distributions whose supports are subsets of R and $S = \left[\underline{s}, \overline{s}\right]$. Lemma C1, a generalization of Lemma 1, characterizes the optimal mechanism.

Lemma C1 *The optimal mechanism is given by*

$$\phi^*(m_1|s,r) = \begin{cases} 1 \text{ if } s > s^*(r), \\ \pi^*(r) \text{ if } s = s^*(r), \\ 0 \text{ if } s < s^*(r). \end{cases} \tag{C.1}$$

If $\int_{\underline{s}}^{\overline{s}} s dF(s|r) \geq u_0$, *then* $s^*(r) = \underline{s}$ *and* $\pi^*(r) = 1$; *otherwise* $s^*(r) \leq 0$ *and* $q^*(r) \equiv \pi^*(r)\Pr(s = s^*(r)|r) \in [0, \Pr(s = s^*(r)|r)]$ *are the unique solution to*

$$\mathbb{E}_{\phi^*}[s - u_0|m_1] = \int_{(s^*(r),\overline{s}]} (s - u_0)\, dF(s|r) + (s^*(r) - u_0)\, q^*(r) = 0. \tag{C.2}$$

Proof. By Fubini's Theorem, the optimal mechanism ϕ^* solves

$$\underset{\phi(m_1|s,r)\in[0,1]}{\text{maximize}} \int_R \left(\int_S \phi(m_1|s,r)\, dF(s|r) \right) dG(r)$$

subject to

$$\int_S (s - u_0)\, \phi(m_1|s,r)\, dF(s|r) \geq 0 \text{ for all } r \in R.$$

We can see that the problem is separable; specifically, for each r, the optimal mechanism ϕ^* maximizes the inside integral subject to the constraint. Therefore, $\phi^*(m_1|s,r) = \widetilde{\phi}^*(m|s)$, where $\widetilde{\phi}^*(m|s)$ is the optimal mechanism in the model in which r is fixed, and the distribution of s is given by $F(s|r)$. Thus, we can omit r from consideration as if it was fixed.

Now I prove that if $\int_{\underline{s}}^{\overline{s}} sdF(s) < u_0$, then the optimal mechanism ϕ^* satisfies (C.1) where (s^*, π^*) solves (C.2). The remaining parts of Lemma C1 are immediate. Suppose to get a contradiction that there exists a mechanism $\widetilde{\phi}$ that results in a higher probability that the receiver acts: $\Pr_{\widetilde{\phi}}(m_1) > \Pr_{\phi^*}(m_1)$. In the next paragraph, I show that $F_{\phi^*}(s|m_1) \leq F_{\widetilde{\phi}}(s|m_1)$ for all $s \in [\underline{s}, \overline{s}]$ with strict inequality for $s \in [s^*, \overline{s})$, and, thus, $\mathbb{E}_{\phi^*}[s|m_1] > \mathbb{E}_{\widetilde{\phi}}[s|m_1]$ by the well-known result (a strong version of Theorem 1 part 1). Therefore, $\mathbb{E}_{\widetilde{\phi}}[s|m_1] < u_0$ because $\mathbb{E}_{\phi^*}[s|m_1] = u_0$ by (C.2). The conclusion that $\mathbb{E}_{\widetilde{\phi}}[s|m_1] < u_0$ contradicts the assumption that the message m_1 induces the receiver to act.

To complete the proof, I show that $F_{\phi^*}(s|m_1) \leq F_{\widetilde{\phi}}(s|m_1)$ for all $s \in [\underline{s}, \overline{s}]$ with strict inequality for $s \in [s^*, \overline{s})$. The inequality trivially holds for $s < s^*$ because $F_{\phi^*}(s|m_1) = 0$ and for $s = \overline{s}$ because $F_{\phi^*}(\overline{s}|m_1) = F_{\widetilde{\phi}}(\overline{s}|m_1) = 1$. Denote the joint distribution of m and s by $\phi(m, s)$. The following sequence of equalities and inequalities proves that $F_{\phi^*}(s|m_1) < F_{\widetilde{\phi}}(s|m_1)$ for $s \in [s^*, \overline{s})$:

$$\begin{aligned}
1 - F_{\phi^*}(s|m_1) &= \frac{\phi^*(m_1, \overline{s}) - \phi^*(m_1, s)}{\Pr_{\phi^*}(m_1)} \\
&= \frac{F(\overline{s}) - F(s)}{\Pr_{\phi^*}(m_1)} \\
&= \frac{\widetilde{\phi}(m_1, \overline{s}) - \widetilde{\phi}(m_1, s)}{\Pr_{\phi^*}(m_1)} + \frac{\widetilde{\phi}(m_0, \overline{s}) - \widetilde{\phi}(m_0, s)}{\Pr_{\phi^*}(m_1)} \\
&\geq \frac{\widetilde{\phi}(m_1, \overline{s}) - \widetilde{\phi}(m_1, s)}{\Pr_{\phi^*}(m_1)} \\
&> \frac{\widetilde{\phi}(m_1, \overline{s}) - \widetilde{\phi}(m_1, s)}{\Pr_{\widetilde{\phi}}(m_1)} \\
&= 1 - F_{\widetilde{\phi}}(s|m_1).
\end{aligned}$$

The first and last equalities hold by Bayes' rule. The second equality holds by (C.1), which defines $\phi^*(m, s)$. The third equality holds by the consistency condition: $\phi(m_1, s) + \phi(m_0, s) = F(s)$ for all mechanisms ϕ and all $s \in [\underline{s}, \overline{s}]$. The first inequality holds because $\phi(m_0, .)$ is a distribution function of s. The second inequality holds by the assumption that $\Pr_{\widetilde{\phi}}(m_1) > \Pr_{\phi^*}(m_1)$. ■

Proposition 1 holds regardless of whether F_1 and F_2 admit densities. The original proof of the second part of Proposition 1 applies to arbitrary F_1 and F_2. To prove the first part of Proposition 1, one should replace the inverse functions with the quantile functions in Theorem 1 part 3 (c) and in the original proof. Specifically, for an arbitrary distribution P, the quantile function is defined as $\varphi(p) \equiv \inf\{x : p \leq P(x)\}$. If $F_2 \geq_{icx} F_1$, then the receiver acts with a higher probability under F_2 than under F_1 because

$$\int_{F_1(s_1^*)-q_1^*}^{1} \varphi_2(\widetilde{p})\, d\widetilde{p} \geq \int_{F_1(s_1^*)-q_1^*}^{1} \varphi_1(\widetilde{p})\, d\widetilde{p} = \int_{(s_1^*, \overline{s}]} sdF_1(s) + s_1^* q_1^*.$$

Conversely, if $F_2 \ngeq_{icx} F_1$, there exists p such that $\int_p^1 \varphi_2(\widetilde{p})\, d\widetilde{p} < \int_p^1 \varphi_1(\widetilde{p})\, d\widetilde{p}$, so the receiver acts with a strictly higher probability under F_1 than under F_2 if $u_0 = \int_p^1 \varphi_2(\widetilde{p})\, d\widetilde{p} / (1 - p)$. Using similar logic, it is straightforward to extend all results to the case of arbitrary distribution functions.

Appendix D. Discussion of Proposition 3

Proposition 3 extends Proposition 2 to the general case of continuous s. Since r is multidimensional in this case, the proposition relies on multidimensional stochastic orders presented in Appendix A (see Definition 2 and Theorem 2). For first-order stochastic dominance and any other stochastic order based on it, we need to introduce a partial order on R. In the case of binary s, the set R is the unit interval [0, 1], a totally ordered set. But what order can we impose on R when R is the set of distributions on $[\underline{s}, \overline{s}]$? To answer this question, consider two degenerate distributions G_1 and G_2 that assign probability 1 to $r_1 = F_1$ and $r_2 = F_2$, respectively. Proposition 1 implies that the sender and receiver are better off under G_2 if $F_2 \geq_{icx} F_1$. To be able to compare such G_1 and G_2, Proposition 3 uses an increasing convex order as a partial order on R.

We lose necessity in Proposition 3 because an increasing convex order is not a total order when s takes more than two values. In part 2 of Proposition 3, we can actually use a total order on R and regain necessity. By Lemma 1, only the distribution of $\mathbb{E}[s|r]$ matters for the receiver. Identifying r with $\mathbb{E}[s|r]$, we obtain the following result. The receiver's expected utility under the optimal mechanism is higher under G_2 than under G_1 for all u_0 if and only if $G_2 \geq_{icx} G_1$, as follows from Theorem 1 part 3 (b) and

$$U_R = \int_R \max\{u_0, \mathbb{E}[s|r]\}\, dG(r) = \int_{\underline{r}}^{\overline{r}} \max\{u_0, r\}\, dG(r) = \overline{r} - \int_{u_0}^{\overline{r}} G(r)\, dr.$$

For the results concerning the sender's expected utility, we must use an increasing convex order on R because of Proposition 1 part 1. But the fact that the sender's expected utility is higher under G_2 than under G_1 for all u_0 does not imply that $G_2 \geq_{micv} G_1$, so necessity cannot be regained. To see this, consider the following counterexample. Let G_1 assign probabilities $(2/3, 1/3, 0)$ to $\left(r^A, r^B, r^C\right)$, and G_2 assign probabilities $(0, 0, 1)$ to $\left(r^A, r^B, r^C\right)$ where r^A assigns probabilities $\left(r^A_1, r^A_2, r^A_3\right) = (0, 1/2, 1/2)$ to $(s_1, s_2, s_3) = (0, 1/2, 1)$, r^B assigns probabilities $(0, 7/8, 1/8)$ to (s_1, s_2, s_3), and r^C assigns probabilities $(3/8, 0, 5/8)$ to (s_1, s_2, s_3).

By Lemma C1, the receiver acts with probability $\min\left\{(4u_0 - 2)^{-1}, 1\right\}$ under r^A, with probability $\min\left\{(16u_0 - 8)^{-1}, 1\right\}$ under r^B, and with probability $\min\left\{(8u_0/5)^{-1}, 1\right\}$ under r^C. By considering all cases ($u_0 \leq 9/16$, $9/16 < u_0 \leq 5/8$, $5/8 < u_0 \leq 3/4$, $3/4 < u_0 \leq 1$, and $u_0 > 1$), it is straightforward to check that the sender's expected utility is always higher under G_2 than under G_1.

By Theorem 1 part 3 (b), for any r and r' supported on (s_1, s_2, s_3), we have $r \geq_{icx} r'$ if and only if $r_3 \geq r'_3$ and $r_2 s_2 + r_3 s_3 \geq r'_2 s_2 + r'_3 s_3$. Thus, the function $h(r) = 5(r_2 + 2r_3) + r_3$ is increasing in r in the increasing convex order. Moreover, h is concave in r because it is linear in r. However, the expectation of h is strictly higher under G_1 than under G_2, which implies that $G_2 \not\geq_{micv} G_1$.

Appendix E. Type-dependent preferences

This appendix allows the sender's utility to depend not only on action a but also on type s in a linear way. Specifically, assume that the sender's utility is $((1-\rho)\overline{s} + \rho s - u_0) \cdot a$, where $\rho \in (0, 1)$. Notice that $\rho = 0$ corresponds to type-independent preferences studied in the paper and $\rho = 1$ corresponds to perfectly aligned preferences of the sender and receiver.

Under this specification, the sender's expected utility under the optimal mechanism continues to be monotonic in the distributions of the sender's and public types both in the first-order stochastic dominance sense and the mean-preserving spread sense. But the receiver's expected utility is monotonic only in the distribution of public type and only in the mean-preserving spread sense. Moreover, some of these results are no longer characterization results in that necessity parts are lost.

The optimal mechanism either leaves no rent to the receiver as in Lemma 1 or gives the first-best outcome to the sender in that the receiver acts if and only if $(1-\rho)\overline{s} + \rho s - u_0 \geq 0$, which is equivalent to $s \geq s^{**} \equiv (u_0 - (1-\rho)\overline{s})/\rho$.

Lemma E2 *The optimal mechanism is given by*

$$\phi^*(m_1|s, r) = \begin{cases} 1 \text{ if } s \geq s^*(r), \\ 0 \text{ if } s < s^*(r). \end{cases}$$

If $s^{**} \leq \underline{s}$ *and* $\mathbb{E}_F[s|r] \geq u_0$, *then* $s^*(r) = \underline{s}$; *if* $s^{**} > \underline{s}$ *and* $\mathbb{E}_F[s - u_0|r, s \geq s^{**}] \geq 0$, *then* $s^*(r) = s^{**}$; *otherwise* $s^*(r) \in \left(\underline{s}, u_0\right)$ *is the unique solution to* $\mathbb{E}_F[s - u_0|r, s \geq s^*(r)] = 0$.

Proof. The optimal mechanism ϕ^* solves

$$\underset{\phi(m_1|s,r)\in[0,1]}{\text{maximize}} \int_{S\times R} ((1-\rho)\overline{s} + \rho s - u_0) f(s|r)\, g(r)\, \phi(m_1|s, r)\, dr ds$$

subject to

$$\int_S (s-u_0) f(s|r)\,\phi(m_1|s,r)\,ds \geq 0 \text{ for all } r \in R$$

where the objective function is the sender's expected utility and the constraint requires that the receiver prefers to act whenever he receives m_1.

The Lagrangian for this problem is given by:

$$\mathcal{L} = \int_{S\times R} \left((1-\rho)\,\overline{s} + \rho s - u_0 + \lambda(r)(s-u_0)\right) f(s|r)\,g(r)\,\phi(m_1|s,r)\,drds,$$

where $\lambda(r)\,g(r)$ is a multiplier for the constraint. Since the choice variable $\phi(m_1|s,r)$ belongs to the unit interval, we have $\phi(m_1|s,r) = 1$ if $s \geq ((1+\lambda(r))\,u_0 - (1-\rho)\,\overline{s})\,/\,(\rho+\lambda(r))$ and $\phi(m_1|s,r) = 0$ otherwise where $\lambda(r)$ is 0 if $\int_{\max\{\underline{s},(u_0-(1-\rho)\overline{s})/\rho\}}^{\overline{s}} (s-u_0) f(s|r)\,ds \geq 0$ and is such that the constraint is binding otherwise. ■

The sender's expected utility under the optimal mechanism is monotone in F such that parts 1 of Proposition 1 and Corollary 1 continue to hold.

Proposition E1 *Let F_1 and F_2 be two distributions of s that do not depend on r. The sender's expected utility under the optimal mechanism is higher under F_2 than under F_1 for all u_0 if and only if F_2 is higher than F_1 in the increasing convex order.*

Proof. Let s_i^* be given by Lemma E2 where F is replaced with F_i. The sender's expected utility under F_i is given by

$$U_S = (1-\rho)(\overline{s}-u_0)(1-F_i(s_i^*)) + \rho \int_{s_i^*}^{\overline{s}} (s-u_0)\,dF_i(s)$$

If $F_2 \geq_{icx} F_1$ (see Definition 1), then the sender can achieve a higher expected utility under F_2 than under F_1 because

$$\int_{F_2^{-1}(F_1(s_1^*))}^{\overline{s}} (s-u_0)\,dF_2(s) = \int_{F_1(s_1^*)}^{1} \left(F_2^{-1}(\widetilde{p}) - u_0\right) d\widetilde{p} \geq \int_{F_1(s_1^*)}^{1} \left(F_1^{-1}(\widetilde{p}) - u_0\right) d\widetilde{p} = \int_{s_1^*}^{\overline{s}} (s-u_0)\,dF_1(s) \geq 0,$$

where the equalities hold by the appropriate change of variables, the first inequality holds by Theorem 1 part 3 (c), and the last inequality holds by Lemma 1. Conversely, if $F_2 \not\geq_{icx} F_1$, then by Theorem 1 part 3 (c), there exists p such that $\int_p^1 F_2^{-1}(\widetilde{p})\,d\widetilde{p} < \int_p^1 F_1^{-1}(\widetilde{p})\,d\widetilde{p}$. Since s_2^* is continuous in u_0, $s_2^* = \underline{s}$ at $u_0 = \underline{s}$, and $s_2^* = \overline{s}$ at $u_0 = \overline{s}$, the mean value theorem implies that there exists u_0 such that $F_2\left(s_2^*\right) = p$ at u_0. Using an analogous argument, we get that the sender achieves a strictly higher expected utility under F_1 than under F_2 at this u_0:

$$\int_{F_1^{-1}(p)}^{\overline{s}} (s-u_0)\,dF_1(s) = \int_p^1 \left(F_1^{-1}(\widetilde{p}) - u_0\right) d\widetilde{p} > \int_p^1 \left(F_2^{-1}(\widetilde{p}) - u_0\right) d\widetilde{p} = \int_{s_2^*}^{\overline{s}} (s-u_0)\,dF_2(s) \geq 0.$$

■

Similarly, fixing the prior distribution of the states as in Corollary 1, it is straightforward to show that the sender's expected utility is higher under one information structure than under another one for all values of u_0 if and only if the latter is a garbling of the former.

However, the receiver's expected utility under the optimal mechanism changes ambiguously with F; so parts 2 of Proposition 1 and Corollary 1 no longer hold. For example, let $u_0 = 0$, $\rho = 2/3$, and consider two distributions: (i) F_1 assigns probability $2/3$ to $s = -2$ and probability $1/3$ to $s = 2$, and (ii) F_2 assigns equal probabilities to $s = -4-x$, $s = x$, and $s = 2$, where $x \in (-2,2)$. Under both distributions, $s^*(r) = s^{**} = -1$, because $(1/3)\cdot 2 > 0$ and $(1/3)\cdot x + (1/3)\cdot 2 > 0$. F_1 is clearly a garbling of F_2, but the receiver's expected utility is higher under F_2 if $x > 0$ and higher under F_1 if $x < 0$.

The sender's expected utility also increases in G in the increasing concave order but the receiver's expected utility increases in G only in the convex order; so Corollary 3 and part 1 of Proposition 3 continue to hold, but part 2 of Proposition 3 no longer holds.

Proposition E2 *Let R be the set of distributions on $\left[\underline{s}, \overline{s}\right]$ and let G_1 and G_2 be two distributions of r.*

1. *Let R be endowed with an increasing convex order. The sender's expected utility under the optimal mechanism is higher under G_2 than under G_1 for all u_0 if G_2 is higher than G_1 in the increasing concave order.*
2. *The receiver's expected utility under the optimal mechanism is higher under G_2 than under G_1 for all u_0 if G_2 is higher than G_1 in the convex order.*

Proof. The sender's expected utility is $\int_R U_S^*(r)\, dG(r)$ where $U_S^*(r)$ is the conditional expected utility. The function U_S^* is increasing in r in the increasing convex order by Proposition E1. Moreover, U_S^* is concave in r, as I show in Proposition 3. Therefore, part 1 of the proposition follows by Theorem 2 part 4.

The receiver's expected utility under the optimal mechanism is

$$U_R = \int_R \max\left\{u_0, u_0 + \int_{s^{**}}^{\overline{s}} (s - u_0)\, dF(s|r), \int_{\underline{s}}^{\overline{s}} s dF(s|r)\right\} dG(r).$$

All three functions inside of the maximum are linear in r because the integrals are linear in $F(.|r)$, which is identified with r. The maximum of the three linear functions is convex in r. Therefore, Part 2 of the proposition follows by Theorem 2 part 2. ■

Notice that part 2 of the proposition holds even if the sender's utility is $u(s) \cdot a$ where u is an arbitrary continuous function of s. Indeed, in this case, the optimal mechanism again either leaves no rent to the receiver or delivers the first best outcome to the sender in that the receiver acts if and only if $s \in S^{**} = \{s : u(s) \geq 0\}$. Then the only change in the expression for U_R in the proof of Proposition E2 is that the middle function inside of the maximum has integration over S^{**}. Since S^{**} does not depend on r, the middle function is still linear in r and the proof goes through.

Finally, the sufficiency parts of Proposition 2 and Corollary 2 hold, but the necessity parts no longer hold. Indeed, if $u_0 \geq (1-\rho)\overline{s} + \rho\underline{s}$, then $s^*(r) = s^{**} > \underline{s}$ and the sender's expected utility is

$$U_S = (\overline{s} - u_0) \int_0^1 r dG(r),$$

but if $u_0 < (1-\rho)\overline{s} + \rho\underline{s}$, then $s^{**} < \underline{s}$ and the sender's expected utility is:

$$\begin{aligned} U_S &= \int_R \min\left\{\begin{array}{c} \frac{\overline{s}-u_0}{u_0-\underline{s}} r\left((1-\rho)(\overline{s}-u_0) + \rho\left(\underline{s}-u_0\right)\right) + r(\overline{s}-u_0), \\ (1-\rho)\overline{s} + \rho\left(\underline{s}r + \underline{s}(1-r)\right) - u_0 \end{array}\right\} dG(r) \\ &= \rho\left(\overline{s}-\underline{s}\right)\int_0^1 r dG(r) + \left((1-\rho)\overline{s} + \rho\underline{s} - u_0\right)\left(1 - \frac{\overline{s}-\underline{s}}{u_0-\underline{s}}\int_0^{\frac{u_0-\underline{s}}{\overline{s}-\underline{s}}} G(r)\, dr\right), \end{aligned}$$

where the first equality holds by Lemma C1 and convention $\Pr(\overline{s}|r) = r$ and the second by integration by parts. Notice that the upper bound $\left(u_0 - \underline{s}\right) / \left(\overline{s} - \underline{s}\right)$ in the last integral varies between 0 and $1-\rho < 1$ as u_0 varies between $\underline{s}$ and $(1-\rho)\overline{s} + \rho\underline{s}$. Therefore, the sender's expected utility may be higher under G_2 for all u_0 even though $\int_0^x G_2(r)\, dr$ is higher than $\int_0^x G_1(r)\, dr$ for some $x \in (1-\rho, 1)$, which implies that G_2 is not higher than G_1 in the increasing concave order by Theorem 1 part 4 (b). A similar argument shows that the receiver's expected utility may be higher under G_2 for all u_0 even though G_2 is not higher than G_1 in the increasing convex order.

Appendix F. Receiver's verifiable private information

In this appendix, the receiver has verifiable private information at the ex ante stage. As usual, verifiable information is the information that cannot be lied about but can be concealed. In this case, the sender extracts the receiver's information at no cost and then discloses her information optimally as if the receiver's type was public. Therefore, all results of Section 3 apply.

To illustrate this result, assume that type r is privately known by the receiver rather than publicly known. In other respects, the environment is the same as in Section 2. In particular, players, actions, the information structure, and preferences are the same. In addition, assume that the set of receiver's types R is given by $[\underline{r}, \overline{r}]$ and is ordered in such a way that $s^*(r)$ is strictly increasing in r where $s^*(r)$ is given by Lemma 1.

Assume that the set of receiver's reports is $N(r) = [\underline{r}, r]$. That is, the receiver can report any type that is lower than his true type. Intuitively, the report n can be viewed as the receiver's claim that his true type r is at least n and the receiver's claims are required to be truthful in that r must belong to $[n, \overline{r}]$.

Now a mechanism ϕ sends a message m to the receiver as a (stochastic) function of (s, n). Finally, the timing of the game is as follows: 1. The sender publicly chooses a mechanism $\phi(m|s, n)$. 2. The receiver's type r is drawn according to G. 3. The receiver makes a report n. 4. A pair (m, s) is drawn according to ϕ and F. 5. The receiver gets a message m and takes an action a. 6. Utilities are realized.

Again, the solution concept used is PBE. In the unique PBE, the receiver discloses all her verifiable private information, as Proposition F3 shows.

Proposition F3 *In the unique PBE, the receiver reports his true type $n = r$ and the sender chooses the optimal mechanism ϕ^* given by Lemma 1.*

The proof of existence of fully revealing equilibrium is by construction. To show the uniqueness, I construct a mechanism which is arbitrarily close to ϕ^* and which makes the receiver strictly prefer to disclose his information.

Proof. I start by showing that the described strategies constitute a PBE. If the receiver reports $n = r$, then his interim expected utility is $\max\{u_0, \mathbb{E}[s|r]\}$ as follows from Lemma 1. If the receiver reports $n < r$, then his interim expected utility is again $\max\{u_0, \mathbb{E}[s|r]\}$ because $s^*(r)$ is increasing in r. Thus, given the mechanism ϕ^*, it is a best response for the receiver to report his true type $n = r$. To see that it is optimal for the sender to choose ϕ^* at the first stage, note that ϕ^* is the optimal mechanism in the relaxed problem where r is publicly known, so ϕ^* gives a higher expected utility to the sender than any other feasible mechanism.

To complete the proof, I show that in all PBEs, the sender chooses ϕ^* and the receiver reports $n = r$. Suppose to get a contradiction that there exists another PBE. In this PBE, the sender's expected utility is strictly less than in the above PBE because ϕ^* is the optimal mechanism in the relaxed problem. Consider a mechanism $\widetilde{\phi}$ that sends the message m_1 if and only if $s \geq s^*(r) + \delta$ where $\delta > 0$ is sufficiently small. Under this mechanism, the receiver strictly prefers to report his true type r and the sender's expected utility is arbitrarily close to that under ϕ^*. A contradiction. ■

Note that the mechanism ϕ^* and truthful reporting of the receiver constitutes a PBE even if the sender has partial commitment in that she can choose a mechanism only after the receiver's report. However, this PBE is not unique in this new model. For example, there exists a PBE in which the receiver always reports $n = 0$. Indeed, suppose that the sender believes that each out-of-equilibrium report $n \neq 0$ is made by the receiver with type $r = n$. Note that under such a belief, the sender chooses a mechanism $\phi^*(m|s, n)$ for any $n \neq 0$. Thus, the receiver's interim expected utility from reporting $n \neq 0$ is $\max\{u_0, \mathbb{E}[s|r]\}$, which is smaller than that from reporting $n = 0$.

Appendix G. Sender's verifiable private information

In this appendix, the sender has verifiable private information before she chooses a mechanism. As a result, the sender discloses all of this information. Thus, without loss of generality, the sender's verifiable information can be viewed as public information. Again, all results of Section 3 apply.

Assume that type r is privately known by the sender rather than publicly known. In other respects, the environment is the same as in Section 2. In addition, assume that R is given by $[\underline{r}, \overline{r}]$ and is ordered in such a way that $s^*(r)$ is strictly decreasing in r. Finally, the sender's information is verifiable in that the set of her reports is $N(r) = [\underline{r}, r]$, where the report n can be viewed as the sender's truthful claim that her type is at least n.

The timing of the game is as follows: 1. The sender's type r is drawn according to G. 2. The sender makes a report n. 3. The sender publicly chooses a mechanism $\phi(m|s, n)$. 4. A pair (m, s) is drawn according to ϕ and F. 5. The receiver gets a report n and a message m and takes an action a. 6. Utilities are realized.

Again, the solution concept is PBE. In the unique PBE, the sender discloses all her verifiable private information, as Proposition G4 shows.

Proposition G4 *In the unique PBE, the sender reports her type truthfully, $n = r$, and chooses the optimal mechanism ϕ^* given by Lemma 1.*

Proof. Suppose to get a contradiction that there exists a report n and a mechanism ϕ_n such that types $r \in \left[\underline{r}_n, \overline{r}_n\right]$ with $\overline{r}_n > \underline{r}_n$ send n and choose ϕ_n with strictly positive probability. Let G_n denote the receiver's belief about r upon receiving n and observing ϕ_n. By definition of m_1,

$$\int_R \int_S (s - u_0) f(s|r) \phi_n (m_1|s, r) ds dG_n(r) \geq 0.$$

Since the sender r does not want to deviate and reveal her type, we have

$$\int_S (s - u_0) f(s|r) \phi_n (m_1|s, r) ds \leq 0 \text{ for all } r \in \left[\underline{r}_n, \overline{r}_n\right].$$

Taking into account the first inequality gives that the second inequality must hold with equality for all $r \in \left[\underline{r}_n, \overline{r}_n\right]$. Therefore, $\phi_n (m_1|s) = \phi^* (m_1|s, r)$ for all $r \in \left[\underline{r}_n, \overline{r}_n\right]$ where ϕ^* is given by Lemma 1, otherwise the sender r would prefer to reveal her type and choose ϕ^*. But this implies that $s^*\left(\underline{r}_n\right) = s^*(\overline{r}_n)$, which contradicts the assumption that s^* is strictly decreasing in r. ■

To conclude, I briefly comment on the case in which the sender's type r is unverifiable in that the set of her reports is $N = [\underline{r}, \overline{r}]$ regardless of r. In this case, the full commitment optimum (ϕ^{**} given by Lemma 1 where $F(s|r)$ is replaced with $\int_R F(s|r) dG(r)$ for all r) can be supported as an equilibrium outcome. Indeed, it is easy to see that there exists a PBE in which for all r the sender makes the same report n and then chooses ϕ^{**}, whereas the receiver believes that the sender's type is $\underline{r}$ for any out-of-equilibrium event. However, there exist many other PBEs, which survives the intuitive criterion. There exist not only pooling but also hybrid PBEs. For example, there may exist a PBE in which types $r < r^*$ choose a mechanism that sends m_1 for $s \in [s', \overline{s}]$ and types $r \geq r^*$ choose a mechanism that sends m_1 for $s \in [s'', s'] \cup [0, \overline{s}]$, where $\underline{s} < s'' < s' < 0$.