ON MONOTONE PERSUASION

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We study monotone persuasion in the linear case, where posterior distributions over states are summarized by their mean. We develop a novel methodological approach to solve two leading cases where optimal unrestricted signals can be nonmonotone. First, if the objective is s-shaped and the state is discrete, then optimal monotone signals are so-called upper censorship, whereas optimal unrestricted signals may require randomization. Second, if the objective is m-shaped and the state is continuous, then optimal monotone signals are so-called interval disclosure, whereas optimal unrestricted signals may require nonmonotone pooling. We illustrate our results with an application to media censorship. Finally, we discuss non-monotone comparative statics and worst-case performance of monotone persuasion.

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1. INTRODUCTION

The literature on Bayesian persuasion has largely focused on the *linear* case, where the state space is one-dimensional and posterior distributions over states are summarized by their mean (e.g., Gentzkow and Kamenica 2016, Kolotilin et al. 2017, Kolotilin 2018, and Dworczak and Martini 2019). The standard approach has been to analyze *unrestricted* persuasion, where the set of feasible signals is unrestricted. In reality, however, various constraints arise due to incentive, legal, or other practical considerations. Two such constraints are that signals should be *deterministic* and *monotone*. For instance, the bank regulator may be unable to use a stress test that credibly and verifiably randomizes scores or gives higher scores to weaker banks (Goldstein and Leitner 2018).[1,2]

These concerns motivate the study of *monotone persuasion* where all feasible signals are deterministic and monotone, so that they partition the state space into convex sets (i.e., intervals and singletons). Dworczak and Martini (2019) delineate conditions under which optimal unrestricted linear persuasion is monotone, so that standard results apply.[3] It remains an open question what optimal monotone signals look like when optimal unrestricted signals are nonmonotone. We answer this question in two leading cases that are most relevant in applications.

The first case is the simplest case where randomization is valuable: the state is discrete, and the objective function is s-shaped (convex-concave). Here, it is known that the optimal unrestricted signal is stochastic upper censorship that separates low states, pools high

[1]See Onuchic and Ray (2023) for other justifications of monotone persuasion. Relatedly, Myerson (1981) and Laffont and Martimort (2009, p. 67) justify monotonicity and nonrandomization constraints in mechanism design.

[2]The Bayesian persuasion literature has considered other types of constraints whose effects differ substantially from the monotonicity constraint. Ivanov (2021), Aybas and Turkel (2026), Hopenhayn and Saeedi (2026), and Lyu et al. (2026) study persuasion under the constraint that a set of signal realizations is finite, whereas Doval and Skreta (2024) study persuasion under a finite number of linear constraints.

[3]Other papers touch upon linear monotone persuasion. Mensch (2021) studies a linear subcase where the objective is quadratic and the solution is trivial: full or no disclosure is optimal. Onuchic and Ray (2023) study a nonlinear case that coincides with the linear case only when all signals are optimal. Farther afield, Arieli et al. (2023) show that, if the state is continuous, it is without loss of optimality to restrict attention to (possibly nonmonotone) deterministic signals or to (possibly stochastic) signals such that a higher state induces a higher lottery over signal realizations with respect to first-order stochastic dominance. More generally, Kolotilin and Zapechelnyuk (2025) show that, regardless of whether the state is discrete or continuous, it is without loss of generality to restrict attention to (possibly stochastic) signals such that a higher state induces a higher lottery over signal realizations with respect to the likelihood ratio order.

states, and randomizes between separation and pooling at the cutoff state. We show that any optimal monotone signal is deterministic upper censorship with the same cutoff state.

The second case is the simplest case where nonmonotone pooling of states is valuable: the state is continuous, and the objective function is m-shaped (concave-convex-concave).[4] If optimal unrestricted signals are nonmonotone, then they induce two signal realizations that concavify the objective.[5] We show that any optimal monotone signal in this case is either no disclosure or a cutoff rule that reveals whether the state is below or above a cutoff.

Our main contribution is to develop a novel methodological approach to monotone persuasion. This approach is quite general and applies to our two leading cases. Our key step narrows down the set of possible optimal monotone signals to a simple class by showing that any monotone signal outside of this class is dominated by a signal in this class. In contrast, existing approaches from the persuasion literature – such as concavification and linear programming duality – do not apply because the monotone persuasion problem is not a linear program.

To illustrate the relevance of our two cases, we use our results to obtain novel economic insights in the media censorship model of Kolotilin et al. (2022), which features a government, heterogeneous citizens, and media outlets. They assume that initially there is a continuum of media outlets and the distribution of citizens' types is unimodal. For any initial set of media outlets, we show that the government's problem of media censorship reduces to a monotone persuasion problem. Our first case corresponds to the case where there is initially a finite number of media outlets and the distribution of citizens' types is unimodal. In this case, the government permits all sufficiently supportive outlets and censors all other media outlets, which extends the result of Kolotilin et al. (2022) on the optimality of upper censorship from the continuous case to the discrete one. Our second case corresponds to the case where there is initially a continuum of media outlets and the distribution of citi-

[4]Various settings feature m-shaped objectives: e.g., the investment recommendation application in Dworczak and Martini (2019, Section 6.2), the relational communication framework of Kolotilin and Li (2021), and the media censorship application discussed in Section 5.

[5]If the optimal unrestricted signal is monotone, then it is interval disclosure which pools low states, separates middle states, and pools high states.

zens' types is bimodal (i.e., society is polarized).[6] In this case, we obtain a novel result: the government either censors all outlets or permits only one moderate outlet. In both cases, the optimal media censorship policies are simple and intuitive. In contrast, optimal unrestricted forms of media control are often too complex and unrealistic.

Our analysis of optimal monotone persuasion yields two novel implications. First, unlike with unrestricted persuasion, a less informative prior distribution of the state can increase the value and informativeness of the optimal monotone signal. Second, while monotone persuasion guarantees $1/2$ of the value of unrestricted persuasion if the state is continuous, it cannot guarantee any positive ratio if the state is discrete.

2. MODEL

A *state* $\omega \in [0,1]$ is a random variable with a prior probability distribution function F. A *signal* is also a random variable, which reveals information about the state. An *objective* $V : [0,1] \mapsto \mathbb{R}$ is a twice continuously differentiable function of the expected state m induced by a signal.

In many applications, the state is either continuous or discrete. The state is *continuous* if F has a strictly positive density f on $[0,1]$. The state is *discrete* if the support of F, denoted by $\operatorname{supp}(F)$, is a finite subset of $[0,1]$. The discrete density is also denoted by f.

In an *unrestricted persuasion problem*, a signal can be arbitrarily correlated with the state. By Blackwell's informativeness theorem, there exists a signal that induces a probability distribution G of the expected state m iff the prior distribution F is a mean-preserving spread of G (e.g., Kolotilin 2018). Thus, the unrestricted persuasion problem is to find an *optimal unrestricted signal* that maximizes $\int_0^1 V(m)\mathrm{d}G(m)$ over distributions G such that F is a mean-preserving spread of G.

In a *monotone persuasion problem*, a signal is required to be *monotone*: it pools the states into convex sets (i.e., intervals and singletons) and reveals which set contains the realized state. Formally, a monotone signal is an increasing function $\mu : [0,1] \mapsto [0,1]$. W.l.o.g., we identify each signal realization m with the expected state induced by this realization,

[6] A recent literature – reflecting growing interest in the topic of political polarization – provides empirical evidence and theoretical mechanisms for polarization; see, e.g., Andreoni and Mylovanov (2012), Baliga et al. (2013), Fryer et al. (2019), Callander and Carbajal (2022), Bowen et al. (2023), and Boxell et al. (2024).

so $m = \mathbb{E}[\omega|\mu(\omega) = m]$. Thus, the monotone persuasion problem is to find an *optimal monotone signal* that maximizes $\int_0^1 V(\mu(\omega))\mathrm{d}F(\omega)$ over monotone signals μ.

Kamenica and Gentzkow (2011) show that full disclosure (resp., no disclosure) is an optimal unrestricted signal, and thus an optimal monotone signal, if the state is discrete and the objective function is convex (resp., concave). Dworczak and Martini (2019) show that an optimal unrestricted signal is monotone if the state is continuous and the objective function is *affine closed*. In particular, V is affine closed if it has no m-shaped (concave-convex-concave) region.

We study two leading cases where an optimal unrestricted signal may be nonmonotone. In Section 3, the objective is s-shaped (convex-concave) but the state is discrete. In Section 4, the state is continuous but the objective is m-shaped (concave-convex-concave).

3. DISCRETE STATE AND S-SHAPED OBJECTIVE

In this section, the state is discrete and the objective is s-shaped. The objective function V is *s-shaped* if there exists $0 < \omega_M < 1$ such that V is strictly convex on $[0, \omega_M]$ and strictly concave on $[\omega_M, 1]$.[7]

A signal is *stochastic upper censorship* if there exist $\omega^* \in \operatorname{supp}(F)$ and $q^* \in [0, 1]$ such that states in $[0, \omega^*)$ are separated, states in $(\omega^*, 1]$ are pooled, and state ω^* is separated with probability q^* and pooled with probability $1 - q^*$. Let

$$m^* = \frac{\omega^*(1-q^*)f(\omega^*) + \sum_{\omega>\omega^*} \omega f(\omega)}{(1-q^*)f(\omega^*) + \sum_{\omega>\omega^*} f(\omega)}$$

be the expected state conditional on the pooling signal realization. A stochastic upper-censorship signal with (ω^*, q^*) is *deterministic upper censorship* if $q^* \in \{0, 1\}$. This is the

[7]To simplify the exposition, we impose rather strong regularity assumptions, such as continuous differentiability and strict convexity or concavity. Using standard approximation arguments, it is straightforward to obtain weaker versions of all our results in the paper under weaker assumptions. For example, suppose that V is (weakly) convex on $[0, \omega_M]$ and (weakly) concave on $[\omega_M, 1]$. Then there exists an optimal monotone signal that is upper censorship. Indeed, consider $V_\varepsilon(m) = V(m) - \varepsilon(m - \omega_M)^3$ converging uniformly to $V(m)$ as $\varepsilon \downarrow 0$. Since V_ε is strictly convex on $[0, \omega_M]$ and strictly concave on $[\omega_M, 1]$, any optimal monotone signal μ_ε is upper censorship. Taking a subsequence if necessary, μ_ε converges to an upper censorship monotone signal μ. Then μ is optimal under V by the uniform convergence theorem.

monotone signal μ given by

$$\mu(\omega) = \begin{cases} \omega, & \omega \in [0, \omega^*), \\ \omega^*, & \omega = \omega^* \text{ and } q^* = 1, \\ m^*, & \omega = \omega^* \text{ and } q^* = 0, \\ m^*, & \omega \in (\omega^*, 1], \end{cases}$$

Alonso and Câmara (2016) and Kolotilin et al. (2022) show that there exist unique $\omega^* \in \text{supp}(F)$ and $q^* \in [0,1]$ satisfying

$$V(m^*) + V'(m^*)(\omega^* - m^*) \geq V(\omega^*), \quad \text{with equality if } (\omega^*, q^*) \neq (0,0), \tag{1}$$

such that the optimal unrestricted signal is stochastic upper censorship with (ω^*, q^*). Condition (1) is the first-order necessary condition for optimality, which holds with equality at interior (ω^*, q^*) and holds with inequality at boundary $(\omega^*, q^*) = (0,0)$.

THEOREM 1: *Suppose the state is discrete and V is s-shaped. Then any optimal monotone signal is deterministic upper censorship. Moreover, if (ω^*, q^*) is given by (1), then any optimal monotone signal is deterministic upper censorship with $(\omega^*, 0)$ or $(\omega^*, 1)$.*

We prove Theorem 1 in two steps. The first step shows that any optimal monotone signal is upper censorship. Intuitively, since an s-shaped V is convex for low states (which favours their separation) and concave for high states (which favours their pooling),[8] it is optimal to separate low states and pool high states, as prescribed by upper censorship. There is a simple linear-programming proof for the case of stochastic upper censorship (see Kolotilin et al. 2022), but this proof cannot be extended to the case of deterministic upper censorship, because the monotone persuasion problem is a discrete optimization problem when the state is discrete. Instead, for each monotone signal that is not upper censorship, our proof constructs a dominating monotone signal that is upper censorship. In particular, in the case

[8]To see that convexity (resp., concavity) of V favours separation (resp., pooling), notice that, in the case with two states $\omega_1 < \omega_2$, separation (resp., pooling) yields $V(\omega_1)f(\omega_1) + V(\omega_2)f(\omega_2)$ (resp., $V(\omega_1 f(\omega_1) + \omega_2 f(\omega_2))$).

of three states $\omega_1 < \omega_2 < \omega_3$, our construction shows that pooling of states ω_1 and ω_2 and separation of state ω_3 is dominated by either full disclosure or no disclosure.

The second step shows that the optimal deterministic upper censorship cutoff coincides with the optimal stochastic upper censorship cutoff. Intuitively, since V is s-shaped, the value of stochastic upper censorship is quasiconcave in (ω^*, q^*) in the lexicographic order $\geq_{\text{lex}}$, where $(\omega_2^*, q_2^*) \geq_{\text{lex}} (\omega_1^*, q_1^*)$ iff $\omega_2^* > \omega_1^*$ or $\omega_2^* = \omega_1^*$ and $q_2^* \geq q_1^*$. Thus, if the optimal unrestricted signal is stochastic upper censorship with (ω^*, q^*), then any optimal monotone signal is deterministic upper censorship with either $(\omega^*, 0)$ or $(\omega^*, 1)$.

Although Theorem 1 seems intuitive and unsurprising, it is subtler than it appears. By analogous appeal to intuition, one might expect that the optimal deterministic signal would also be deterministic upper censorship, but this is not true. For example, fixing $\varepsilon, q \in (0, 1/2)$, suppose that the prior distribution F assigns probabilities $(1-q)/2$, $q/2$, and $1/2$ to states 0, ε, and 1. Suppose that V is such that the optimal unrestricted signal is stochastic upper censorship that separates state 0 with probability q when the prior distribution is uniform over states 0 and 1. In this case, if ε is sufficiently small, the optimal deterministic signal pools states 0 and 1 and separates state ε.[9]

4. CONTINUOUS STATE AND M-SHAPED OBJECTIVE

In this section, the state is continuous and the objective is m-shaped. The objective function V is *m-shaped* if there exist $0 < \omega_L < \omega_R < 1$ such that V is strictly concave on $[0, \omega_L]$, strictly convex on $[\omega_L, \omega_R]$, and strictly concave on $[\omega_R, 1]$.

A monotone signal μ is *interval disclosure* with cutoffs $0 \leq \omega_L^* \leq \omega_R^* \leq 1$ if states in the middle interval $[\omega_L^*, \omega_R^*]$ are separated and states in the left interval $[0, \omega_L^*)$ and in the right interval $(\omega_R^*, 1]$ are pooled, so

$$\mu(\omega) = \begin{cases} m_L^*, & \omega \in [0, \omega_L^*), \\ \omega, & \omega \in [\omega_L^*, \omega_R^*], \\ m_R^*, & \omega \in (\omega_R^*, 1], \end{cases}$$

[9]Intuitively, as $\varepsilon \to 0$, by continuity, the value of this signal converges to the value of optimal stochastic upper censorship, while the value of any of the other 4 deterministic signals is bounded away from the value of optimal stochastic upper censorship.

where

$$m_L^* = \mathbb{E}\big[\omega|\omega \in [0, \omega_L^*]\big] \quad \text{and} \quad m_R^* = \mathbb{E}\big[\omega|\omega \in [\omega_R^*, 1]\big]$$

are the expected states conditional on the pooling signal realizations. A monotone signal μ is a *cutoff rule* with cutoff ω^* if states in the intervals $[0, \omega^*)$ and $(\omega^*, 1]$ are pooled. Finally, a monotone signal μ is *no disclosure* if all states in $[0, 1]$ are pooled. Note that no disclosure is a special case of a cutoff rule, which is, in turn, a special case of interval disclosure.

It is straightforward to obtain the first-order necessary conditions for optimality (see Kolotilin 2018). If interval disclosure with interior cutoffs $0 < \omega_L^* < \omega_R^* < 1$ is optimal, then

$$V(m_L^*) + V'(m_L^*)(\omega_L^* - m_L^*) = V(\omega_L^*), \tag{2}$$

$$V(m_R^*) + V'(m_R^*)(\omega_R^* - m_R^*) = V(\omega_R^*) \tag{3}$$

If a cutoff rule with interior cutoff $\omega^* \in (0, 1)$ is optimal, then

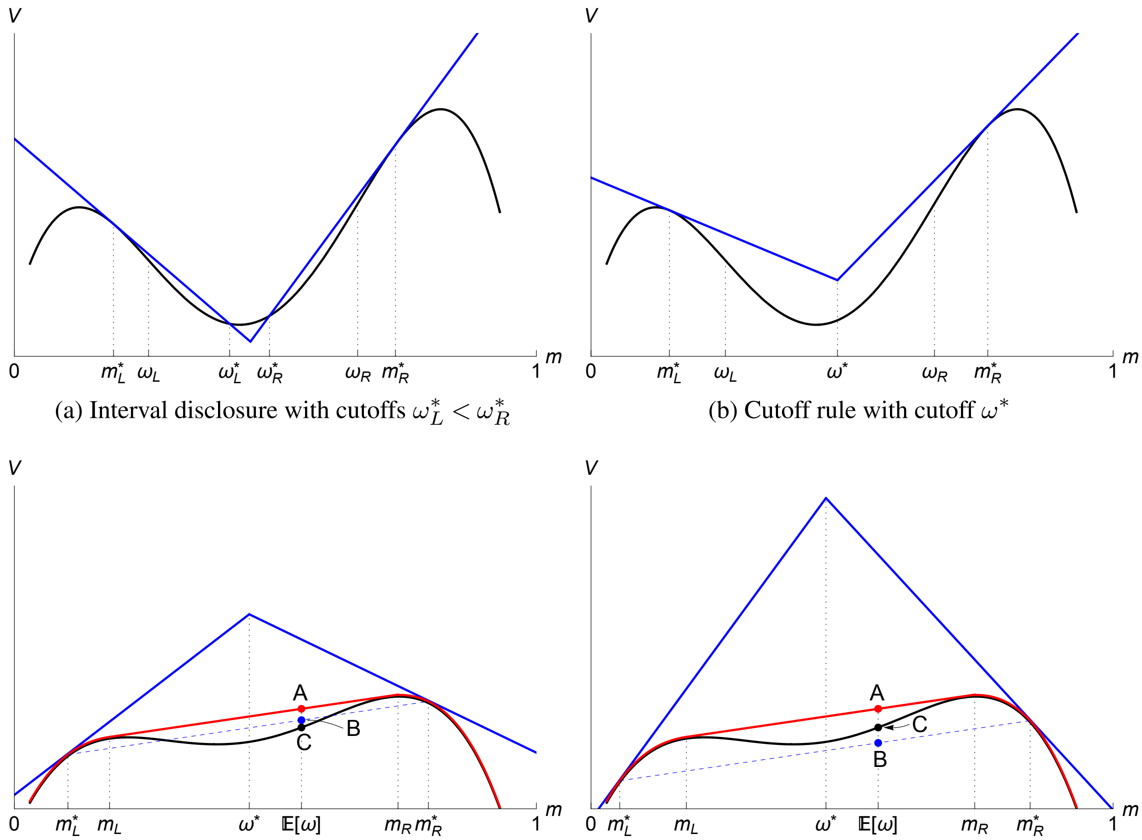
$$V(m_L^*) + V'(m_L^*)(\omega^* - m_L^*) = V(m_R^*) + V'(m_R^*)(\omega^* - m_R^*), \tag{4}$$

where $m_L^* = \mathbb{E}[\omega|\omega \in [0, \omega^*]]$ and $m_R^* = \mathbb{E}[\omega|\omega \in [\omega^*, 1]]$. Also, no disclosure is suboptimal if there exists a cutoff rule with cutoff $\omega^* \in (0, 1)$ such that

$$V(m_L^*)F(\omega^*) + V(m_R^*)(1 - F(\omega^*)) > V(\mathbb{E}[\omega]). \tag{5}$$

THEOREM 2: *Suppose the state is continuous and V is m-shaped. Then any optimal monotone signal is interval disclosure. Moreover:*

1. *If there exist $\omega_L^*, \omega_R^* \in (\omega_L, \omega_R)$ with $\omega_L^* < \omega_R^*$ such that (2) and (3) hold, then the optimal monotone signal is interval disclosure with cutoffs ω_L^* and ω_R^*. Also, $m_L^* \in (0, \omega_L)$ and $m_R^* \in (\omega_R, 1)$.*
2. *Else if there exists $\omega^* \in (0, 1)$ such that (5) holds, then an optimal monotone signal is a cutoff rule with some cutoff $\omega^* \in (0, 1)$ that satisfies (4) and (5). Also, $m_L^* \in (0, \omega_L)$ and $m_R^* \in (\omega_R, 1)$.*
3. *Else, an optimal monotone signal is no disclosure.*



(a) Interval disclosure with cutoffs $\omega_L^* < \omega_R^*$

(b) Cutoff rule with cutoff ω^*

(c) Cutoff rule with cutoff ω^*

(d) No disclosure

FIGURE 1.—Interval disclosure when V is m-shaped.

Note: Moving along Figures 1a → 1b → 1c → 1d, expected states m_L^* and m_R^* move away from each other, which means that the prior distribution F puts increasingly more weight on left and right states (and less weight on middle states). In Figure 1a, the tangents to V at m_L^* and m_R^* cross V at ω_L^* and ω_R^*. In Figures 1b, 1c, and 1d, the tangents to V at m_L^* and m_R^* intersect at ω^*. In Figures 1c and 1d, points A, B, and C show the values of an optimal unrestricted signal, an optimal cutoff rule, and no disclosure.

We prove Theorem 2 in two steps. The first step shows that any optimal monotone signal is interval disclosure. Intuitively, since an m-shaped V is convex for middle states (which favours their separation) and concave for extreme states (which favours their pooling), it is optimal to separate middle states and pool extreme states, as prescribed by interval disclosure. There is a simple linear programming characterization of optimal unrestricted signals when V is m-shaped (see Kolotilin 2018), but all these signals may be nonmonotone. In this case, the standard approaches from the persuasion literature no longer apply. Instead, for each monotone signal that is not interval disclosure, our proof constructs a dominating monotone signal that is interval disclosure.

The second step delineates conditions under which an optimal monotone signal takes each of the three possible forms of interval disclosure: nondegenerate interval disclosure (Figure 1a), a cutoff rule (Figures 1b and 1c), and no disclosure (Figure 1d).

If (2) and (3) hold (Figure 1a), or if (4) and (5) hold and $V'(m_L^*) \leq V'(m_R^*)$ (Figure 1b), then the optimal unrestricted signal is interval disclosure (see Kolotilin 2018). Otherwise (Figures 1c and 1d), each optimal unrestricted signal is nonmonotone. This nonmonotone case arises iff the following condition holds (see Arieli et al. 2023).

CONDITION 1: There is a unique bitangent to V whose tangent points m_L and m_R are such that $0 < m_L < \mathbb{E}[\omega] < m_R < 1$ and there exists $\omega^{**} \in (m_L, 1)$ with $\mathbb{E}[\omega|\omega \in [0, \omega^{**}]] = m_L$ and $\mathbb{E}[\omega|\omega \in [\omega^{**}, 1]] > m_R$.

Condition 1 says that a cutoff rule with cutoff ω^{**} induces expected states $m_L^{**} = m_L$ and $m_R^{**} > m_R$, which intuitively means that the prior distribution F is sufficiently spread out. Kleiner et al. (2021) and Arieli et al. (2023) show that, under Condition 1, each optimal unrestricted signal induces the expected states m_L and m_R and yields the value $\operatorname{co} V(\mathbb{E}[\omega])$, where $\operatorname{co} V(\mathbb{E}[\omega])$ is the concavification of V at $\mathbb{E}[\omega]$,

$$\operatorname{co} V(\mathbb{E}[\omega]) = V(m_L)\frac{m_R - \mathbb{E}[\omega]}{m_R - m_L} + V(m_R)\frac{\mathbb{E}[\omega] - m_L}{m_R - m_L}.$$

An optimal unrestricted signal may be deterministic, but then it is necessarily nonmonotone. For example, there exist $\omega_L^{**} \in (0, m_L)$ and $\omega_R^{**} \in (m_L, 1)$ such that states in $(\omega_L^{**}, \omega_R^{**})$ induce expected state m_L and states in $[0, \omega_L^{**}) \cup (\omega_R^{**}, 1]$ induce expected state m_R. Also, an optimal unrestricted signal may be stochastically monotone (i.e., a higher state induces a higher lottery over expected states with respect to first-order stochastic dominance), but then it is necessarily nondeterministic. For example, there exists $q^{**} \in (0,1)$ such that states in $[0, \omega^{**})$ induce expected states m_L and m_R with probabilities q^{**} and $1 - q^{**}$, and states in $(\omega^{**}, 1]$ always induce expected state m_R.

We find that, under Condition 1, each optimal monotone signal is either a cutoff rule or no disclosure (Parts 2 and 3 of Theorem 2), and it yields a strictly lower value than the optimal unrestricted signal. In particular, the value of a cutoff rule with cutoff ω^*

is $V(m_L^*)F(\omega^*) + V(m_R^*)(1 - F(\omega^*)) < \mathrm{co}\, V(\mathbb{E}[\omega])$, and the value of no disclosure is $V(\mathbb{E}[\omega]) < \mathrm{co}\, V(\mathbb{E}[\omega])$. If (5) holds, then a cutoff rule dominates no disclosure (Figure 1c), and otherwise no disclosure dominates (Figure 1d).

5. APPLICATION TO MEDIA CENSORSHIP

We illustrate our results using the media censorship model of Kolotilin et al. (2022), who characterize optimal media censorship under the following two assumptions. First, there is a continuum of media outlets. Second, the distribution of citizens' types is unimodal. Our results allow us to relax these two assumptions, one at a time. Theorem 1 yields a characterization of an optimal censorship policy when there is a finite (possibly small) number of media outlets. Theorem 2 yields a characterization of an optimal censorship policy when the distribution of citizens' types is bimodal (i.e., society is polarized).

5.1. *Model*

There is a government and a continuum of heterogeneous citizens. The government's quality $\theta \in [0,1]$ has a distribution T with a strictly positive density on $[0,1]$. Citizens are indexed by $r \in [0,1]$ that has a distribution V with a continuously differentiable density on $[0,1]$. The utility of a citizen of type r is

$$u(a_r, \theta, r) = (\theta - r)a_r,$$

where $a_r \in \{0,1\}$ is the citizen's action.[10] The government's utility is the aggregate action in the society $\int_0^1 a_r \mathrm{d}V(r)$.

Citizens receive information about the government's quality θ through media outlets. Each media outlet is identified by its editorial policy $c \in [0,1]$, and it endorses action $a = 1$ if $\theta \geq c$ and endorses action $a = 0$ if $\theta < c$. The set of media outlets C is a subset of $[0,1]$.

[10]Such citizens' preferences can be microfounded by a spatial voting model, as in Chiang and Knight (2011). Consider a government party ($p = G$) and an opposition party ($p = O$) competing in an election. If party p wins, a voter with ideological position r gets utility $w_p - (r - r_p)^2$, where w_p is the valence of party p and r_p is the ideology of party p. Voters know the parties' ideologies but are uncertain about their valences. Each voter supports the party that maximizes his expected utility, where $a_r = 1$ stands for supporting the government party. Our analysis applies, because the voter's utility difference between the government and opposition parties is proportional to $\theta - r$, where $\theta = \left(w_G - w_O + r_O^2 - r_G^2\right)/2(r_O - r_G)$ represents the (normalized) quality of the government.

The government's censorship policy is a set of media outlets $X \subset C$ that are censored. The other media outlets in $C \backslash X$ are permitted to broadcast. The government's problem is to find an *optimal censorship policy* that maximizes its expected utility over censorship policies X.

The timing is as follows. First, the government chooses a set $X \subset C$ of censored media outlets. Second, the government's quality θ is realized, and each permitted media outlet endorses action $a = 1$ or $a = 0$ according to its editorial policy. Finally, each citizen observes messages from all permitted media outlets, updates beliefs about θ, and chooses an action.

Kolotilin et al. (2022) solve the case with $C = [0, 1]$ (i.e., there is a continuum of media outlets) and an s-shaped V (i.e., the distribution of citizens' types is unimodal). If it were possible to design any signal about government's quality θ, then deterministic upper censorship with some cutoff $\theta^* \in [0, 1)$ would be optimal for the government. Thus, it is optimal to censor all media outlets with editorial policies above θ^*, as this censorship policy implements upper censorship with cutoff θ^*. This approach is valid when $C = [0, 1]$ and when the optimal unrestricted signal about θ is monotone. Our results allow us to address more general cases.

5.2. *Reduction to Monotone Persuasion*

We start by showing that the government's problem of media censorship reduces to a monotone persuasion problem with an appropriately defined state.

Consider a censorship policy $X \subset C$. Let y_X be a random variable equal to the conditional expectation of θ given messages from all media outlets in $C \backslash X$. Let G_X denote the distribution of y_X. Each citizen of type r chooses $a_r = 1$ iff $r \leq y_X$. Then, the aggregate action is $\int_0^1 a_r \mathrm{d}V(r) = V(y_X)$, and the government's expected utility is $\int_0^1 V(y)\mathrm{d}G_X(y)$. Let $\mathcal{G}_C$ denote the set of distributions G_X induced by all censorship policies $X \subset C$.

Define the state ω as the conditional expectation of θ given messages from all media outlets in C. That is, $\omega = y_\varnothing$ and its distribution is $F = G_\varnothing$. Consider a monotone signal μ, which is an increasing function satisfying $\mathbb{E}[\omega|\mu(\omega) = m] = m$ for all m. Let G_μ denote the distribution of $m = \mu(\omega)$. Then, the value of μ is $\int_0^1 V(\mu(\omega))\mathrm{d}F(\omega) = \int_0^1 V(m)\mathrm{d}G_\mu(m)$. Let $\mathcal{G}_M$ denote the set of distributions G_μ induced by all monotone signals μ.

The next proposition shows that an outcome is implementable by a monotone signal iff it is implementable by a censorship policy. Thus, the government's problem of media censorship reduces to a monotone persuasion problem.

PROPOSITION 1: $\mathcal{G}_C = \mathcal{G}_M$.

We illustrate the intuition for Proposition 1 using an example with two media outlets whose editorial policies are c_1 and c_2.

EXAMPLE 1: Let $C = \{c_1, c_2\}$ with $0 < c_1 < c_2 < 1$. There are three states, $\omega_1 = \mathbb{E}[\theta|\theta \leq c_1]$, $\omega_2 = \mathbb{E}[\theta|c_1 \leq \theta \leq c_2]$, and $\omega_3 = \mathbb{E}[\theta|\theta \geq c_2]$. There are four censorship policies, (i) $X = \varnothing$, (ii) $X = \{c_1\}$, (iii) $X = \{c_2\}$, and (iv) $X = \{c_1, c_2\}$. They correspond to four monotone signals, (i) full disclosure, (ii) pooling of states ω_1 and ω_2 and separation of state ω_3, (iii) separation of state ω_1 and pooling of states ω_2 and ω_3, and (iv) no disclosure. In particular, no censorship policy implements pooling of states ω_1 and ω_3 and separation of state ω_2.

5.3. *Discrete Unimodal Case*

Suppose that there is a finite number of media outlets (i.e., state ω is discrete), and that the distribution of citizens' types is unimodal (i.e., V is s-shaped). By Theorem 1, the government optimally censors all media outlets whose editorial policies are above some cutoff. That is, all censored media outlets are less supportive than all permitted media outlets in that they endorse the government's preferred action less frequently.

The government may gain from using more sophisticated tools of media control than media censorship. For example, the government may prefer to replace media outlets with one government's media outlet that aggregates information from media outlets, possibly adding random noise. We now show that optimal media control may take intricate forms even when there are only two media outlets.

EXAMPLE 1—continued: The optimal unrestricted signal may take the form of stochastic upper censorship where state ω_2 is separated with probability $q \in (0, 1)$. This signal is implemented by letting media outlet c_1 broadcast freely and by randomly influencing media outlet c_2 as follows. With probability q, media outlet c_2 broadcasts freely. With probability

$1-q$, media outlet c_2 is forced to repeat the message of media outlet c_1. Somewhat unrealistically, citizens must not know if media outlet c_2 is influenced. In turn, the optimal deterministic signal may pool states ω_1 and ω_3 and separate state ω_2. This signal is implemented by letting citizens observe only whether media outlets c_1 and c_2 send the same message or not, without revealing which action this message endorses.

5.4. *Continuous Bimodal Case*

Suppose that there is a continuum of media outlets $C = [0, 1]$ (i.e., state ω is continuous with distribution $F = T$), and that the distribution of citizens' types is bimodal (i.e., society is polarized). For illustration, we restrict attention to the case where V is m-shaped and the distribution of the government's quality is sufficiently spread out in that Condition 1 holds.

The government either optimally censors all media outlets (Part 3 of Theorem 2) or permits only one media outlet with a moderate editorial policy $c^* \in (0, 1)$ (Part 2 of Theorem 2). It may seem counterintuitive that the government optimally censors not only the least supportive media outlets (as in the unimodal case) but also the most supportive ones. Intuitively, the bimodal case corresponds to a polarized society where most citizens are either supporters or opponents, rather than moderates. Thus, censoring most supportive media outlets ensures that supporters continue to choose the government's preferred action even if no permitted media outlets endorse it.

We now discuss two somewhat unrealistic forms of optimal media control, which strictly outperform optimal media censorship under Condition 1.[11] Let $0 < m_L < m_R < 1$, $0 < \omega_L^{**} < \omega^{**} < \omega_R^{**} < 1$, and $q^{**} \in (0, 1)$ be as in Section 4. The first form is deterministic but nonmonotone. Citizens observe only whether media outlets $c_L = \omega_L^{**}$ and $c_R = \omega_R^{**}$ send the same message or not. The second form is stochastically monotone but nondeterministic. Citizens observe the message of only one media outlet $c = \omega^{**}$ that is randomly influenced as follows. With probability q^{**}, media outlet c broadcasts freely. With probability $1 - q^{**}$, media outlet c is forced to endorse the government's preferred action, regardless of the government's quality.

[11] These two forms correspond to the two optimal unrestricted signals discussed in the second to last paragraph in Section 4.

6. IMPLICATIONS OF MONOTONE PERSUASION

Section 6.1 shows that, unlike with unrestricted persuasion, a less informative prior distribution of the state can increase the value of monotone persuasion. Section 6.2 shows that, when the objective V is nonnegative, monotone persuasion guarantees $1/2$ of the value of unrestricted persuasion if the state is continuous, but it cannot guarantee any positive ratio if the state is discrete.

6.1. *Comparative Statics*

We say that a prior $\tilde{F}$ is less informative than a prior F, denoted by $\tilde{F} \preceq F$, if F is a mean-preserving spread of $\tilde{F}$. To interpret this definition, note that our model admits the following microfoundation. Fix a random variable $\theta \in [0,1]$. Consider a decision maker who observes a noisy signal ω about θ and then makes a decision to maximize her utility. Say that a decision maker is linear if her utility depends on the belief about θ induced by ω only through the expected value of θ. W.l.o.g., normalize $\omega = \mathbb{E}[\theta|\omega]$. Given any two signals ω and $\tilde{\omega}$ with distributions F and $\tilde{F}$, all linear decision makers are worse off under $\tilde{F}$ than under F iff $\tilde{F} \preceq F$.[12]

Let $V^U(F)$ and $V^M(F)$ denote the values of optimal unrestricted and monotone persuasion when the prior distribution is F. Specifically, $V^U(F) = \int_0^1 V(m)\mathrm{d}G^U(m)$ and $V^M(F) = \int_0^1 V(m)\mathrm{d}G^M(m)$ where G^U and G^M are the distributions of the expected state induced by optimal unrestricted and monotone signals.

Clearly, a less informative prior distribution cannot increase the value of optimal unrestricted persuasion, because $\tilde{G}^U \preceq \tilde{F}$ by Blackwell's informativeness theorem, so if $\tilde{F} \preceq F$, then $\tilde{G}^U \preceq F$ by transitivity.

In contrast, if optimal unrestricted persuasion is nonmonotone then there always exists a less informative prior distribution that increases the value of optimal monotone persuasion. Indeed, suppose that $V^U(F) > V^M(F)$, and set $\tilde{F} = G^U$. Then $\tilde{F} \prec F$ and $V^M(\tilde{F}) = V^U(\tilde{F}) = V^U(F)$, so $V^M(\tilde{F}) > V^M(F)$. If F is discrete, then $\tilde{F} = G^U$ is also discrete. If F is continuous, then $\tilde{F} = G^U$ is not necessarily continuous, but it is still possible to

[12] Moreover, when θ is binary, $\tilde{F} \preceq F$ iff $\tilde{\omega}$ is Blackwell less informative about θ than ω.

construct a continuous $\tilde{F} \prec F$ such that $V^M(\tilde{F}) = V^U(\tilde{F}) = V^U(F)$ and thus $V^M(\tilde{F}) > V^M(F)$.[13]

This discussion also highlights that a less informative prior distribution of the state can lead to a strictly more informative optimal monotone signal, which cannot occur in unrestricted persuasion. Indeed, in both cases, $G^M \prec \tilde{G}^M$ if $\text{supp}(G^M) = \{\mathbb{E}_F[\omega]\}$, while we cannot have $G^U \prec \tilde{G}^U$, as otherwise both G^U and $\tilde{G}^U$ would be optimal under F, contradicting uniqueness of the optimal distribution of the expected state (Kolotilin 2018).

6.2. *Performance of Monotone Persuasion*

For any $\varepsilon \in (0,1)$, there exist discrete F and nonnegative, s-shaped V such that the value of monotone persuasion $V^M(F)$ is at most ε of the value of unrestricted persuasion $V^U(F)$. Indeed, suppose that F assigns probabilities $1-\varepsilon/2$ and $\varepsilon/2$ to states 0 and 1, and $V(m) = 1$ for $m \in [\varepsilon, 1]$ and $V(m) = 0$ for $m \in [0, \varepsilon)$. The optimal unrestricted signal induces the expected states 0 and ε with equal probabilities, while the optimal monotone signal induces the expected states 0 and 1 with probabilities $1-\varepsilon/2$ and $\varepsilon/2$. Thus, $V^U(F) = 1/2$ and $V^M(F) = \varepsilon/2$.

In contrast, $V^M(F) \geq V^U(F)/2$ for any continuous F and nonnegative V, as shown by Chen et al. (2026). For illustration, suppose that V is m-shaped and $V^U(F) > V^M(F)$. By Theorem 2, there exist $m_L^\dagger < m_L < \mathbb{E}[\omega] < m_R < m_R^\dagger$, $\omega_L^\dagger \in (m_L, 1)$, and $\omega_R^\dagger \in (0, m_R)$ such that $\text{supp}(G^U) = \{m_L, m_R\}$, a cutoff rule with cutoff $\omega_L^\dagger$ induces expected states m_L and $m_R^\dagger$, and a cutoff rule with cutoff $\omega_R^\dagger$ induces expected states $m_L^\dagger$ and m_R. The values of these cutoff rules are at least $V(m_L)F(\omega_L^\dagger) > V(m_L)(m_R - \mathbb{E}[\omega])/(m_R - m_L)$ and $V(m_R)(1 - F(\omega_R^\dagger)) > V(m_R)(\mathbb{E}[\omega] - m_L)/(m_R - m_L)$, because

$$
\begin{aligned}
\mathbb{E}[\omega] &= m_L F(\omega_L^\dagger) + m_R^\dagger(1 - F(\omega_L^\dagger)) > m_L F(\omega_L^\dagger) + m_R(1 - F(\omega_L^\dagger)), \\
\mathbb{E}[\omega] &= m_L^\dagger F(\omega_R^\dagger) + m_R(1 - F(\omega_R^\dagger)) < m_L F(\omega_R^\dagger) + m_R(1 - F(\omega_R^\dagger)).
\end{aligned}
$$

[13]For illustration, let V be m-shaped. By Theorem 2, there exist $m_L^* < m_L < m_R < m_R^*$ such that $\text{supp}(G^U) = \{m_L, m_R\}$ and either $\text{supp}(G^M) = \{m_L^*, m_R^*\}$ (Figure 1c) or $\text{supp}(G^M) = \{\mathbb{E}_F[\omega]\}$ (Figure 1d). But then it is easy to obtain a required $\tilde{F}$ by reallocating the mass from the tails of F to the middle of F such that $\mathbb{E}_{\tilde{F}}[\omega] = \mathbb{E}_F[\omega]$, $\mathbb{E}_{\tilde{F}}[\omega|\omega < \omega^*] = m_L$, and $\mathbb{E}_{\tilde{F}}[\omega|\omega \geq \omega^*] = m_R$ for some ω^*.

Thus, the best of the two cutoff signals guarantees at least $1/2$ of the value of the optimal unrestricted signal

$$V^U(F) = V(m_L)\frac{m_R - \mathbb{E}[\omega]}{m_R - m_L} + V(m_R)\frac{\mathbb{E}[\omega] - m_L}{m_R - m_L}.$$

7. CONCLUSION

A lot of progress has been made on optimal linear persuasion, where posterior distributions over states are summarized by their mean. But little is known about optimal monotone persuasion, beyond when optimal persuasion turns out to be monotone. Optimal persuasion can be nonmonotone when the state is discrete, which requires randomization, or when the objective function is irregular, which requires nonmonotone pooling. We provide two theorems that characterize optimal monotone persuasion in most prominent such cases, so they can be used as off-the-shelf results in follow-up work. Our proofs identify a candidate class of optimal monotone signals and show that any monotone signal outside of this class is dominated by a signal in this class. This approach can be applied more generally by suitably adjusting a candidate class of optimal monotone signals.

We make a case for monotone persuasion in the context of media censorship. But there are many other considerations that lead to monotone persuasion. For example, a nonmonotone grading policy that gives better grades to worse performing students may be viewed as unfair or illegitimate and may be manipulated by strategic students. Moreover, the monotonicity restriction may arise due to Mirrlees incentive constraints (e.g., Rayo 2013 and Kolotilin and Li 2021). Finally, monotone persuasion is equivalent to deterministic delegation (see Kolotilin and Zapechelnyuk 2025), so our results are also relevant for the delegation literature, which has primarily focused on deterministic mechanisms.

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APPENDIX A: PROOFS

A.1. *Proof of Theorem 1*

Let $\operatorname{supp}(F) = \{\omega_1, \ldots, \omega_n\}$, with natural $n \geq 2$ and $\omega_1 < \ldots < \omega_n$. For each $1 \leq i < j \leq n$, denote $f_j = f(\omega_j)$, $f_{i:j} = f_i + \ldots + f_j$, and $m_{i:j} = (\omega_i f_i + \ldots + \omega_j f_j)/f_{i:j}$.

LEMMA 1: *Each optimal monotone signal is deterministic upper censorship.*

PROOF: Suppose by contradiction that there exists an optimal monotone signal μ that is not deterministic upper censorship. Then there exist $1 \leq i < j < k \leq n$ and two signal realizations: $m_{i:j}$ that pools states $\{\omega_i, \ldots, \omega_j\}$ and $m_{j+1:k}$ that pools states $\{\omega_{j+1}, \ldots, \omega_k\}$. Let μ_- and μ_+ be monotone signals that differ from μ only in that μ_- merges signal realizations $m_{i:j}$ and $m_{j+1:k}$ of μ into one signal realization $m_{i:k}$ and μ_+ splits signal realization $m_{i:j}$ of μ into two signal realizations: $m_{i:j-1}$ and ω_j. Denote the value of signals μ_-, μ, and μ_+ by W_-, W, and W_+.

To obtain a contradiction, it suffices to show that $W \geq W_+$ implies $W < W_-$. So, suppose that $W \geq W_+$, which is equivalent to

$$V(m_{i:j}) - \frac{\omega_j - m_{i:j}}{\omega_j - m_{i:j-1}} V(m_{i:j-1}) - \frac{m_{i:j} - m_{i:j-1}}{\omega_j - m_{i:j-1}} V(\omega_j) \geq 0. \tag{6}$$

Since V is strictly convex on $[0, \omega_M]$ and (6) holds, it follows that $\omega_M < \omega_j$.

We now show that $W < W_-$, which is equivalent to

$$V(m_{i:k}) - \frac{m_{j+1:k} - m_{i:k}}{m_{j+1:k} - m_{i:j}} V(m_{i:j}) - \frac{m_{i:k} - m_{i:j}}{m_{j+1:k} - m_{i:j}} V(m_{j+1:k}) > 0. \tag{7}$$

If $\omega_M \leq m_{i:j}$, then (7) follows from strict concavity of V on $[\omega_M, 1]$. So, suppose that $\omega_M \in (m_{i:j}, \omega_j)$. In summary, we have

$$m_{i:j-1} < m_{i:j} < \omega_M < \omega_j < m_{j+1:k}, \quad \text{and} \quad m_{i:j} < m_{i:k} < m_{j+1:k}. \tag{8}$$

Since V is strictly convex on $[0, \omega_M]$ and strictly concave on $[\omega_M, 1]$, by (8), we have

$$\frac{\omega_M - m_{i:j}}{\omega_M - m_{i:j-1}} V(m_{i:j-1}) + \frac{m_{i:j} - m_{i:j-1}}{\omega_M - m_{i:j-1}} V(\omega_M) - V(m_{i:j}) > 0, \tag{9}$$

$$V(\omega_j) - \frac{m_{j+1:k} - \omega_j}{m_{j+1:k} - \omega_M} V(\omega_M) - \frac{\omega_j - \omega_M}{m_{j+1:k} - \omega_M} V(m_{j+1:k}) > 0. \tag{10}$$

$$\frac{m_{i:k} - m_{i:j}}{m_{i:k} - m_{i:j-1}} V(m_{i:j-1}) + \frac{m_{i:j} - m_{i:j-1}}{m_{i:k} - m_{i:j-1}} V(m_{i:k}) - V(m_{i:j}) > 0, \quad \text{if } m_{i:k} \le \omega_M, \tag{11}$$

$$V(m_{i:k}) - \frac{m_{j+1:k} - m_{i:k}}{m_{j+1:k} - \omega_M} V(\omega_M) - \frac{m_{i:k} - \omega_M}{m_{j+1:k} - \omega_M} V(m_{j+1:k}) > 0, \quad \text{if } m_{i:k} \ge \omega_M. \tag{12}$$

If $m_{i:k} \le \omega_M$, then adding the inequalities (6), (9), (10), and (11) multiplied by $(m_{j+1:k} - \omega_M)(m_{i:k} - m_{i:j})(\omega_j - m_{i:j-1})$, $(m_{j+1:k} - \omega_j)(m_{i:k} - m_{i:j})(\omega_M - m_{i:j-1})$, $(m_{i:j} - m_{i:j-1})(m_{i:k} - m_{i:j})(m_{j+1:k} - \omega_M)$, and $(\omega_j - \omega_M)(m_{j+1:k} - m_{i:j})(m_{i:k} - m_{i:j-1})$, respectively, yields (7). If $m_{i:k} \ge \omega_M$, then adding the inequalities (6), (9), (10), and (12) multiplied by $(m_{j+1:k} - m_{i:k})(\omega_j - m_{i:j-1})(\omega_M - m_{i:j})$, $(m_{j+1:k} - m_{i:k})(\omega_M - m_{i:j-1})(\omega_j - m_{i:j})$, $(m_{j+1:k} - m_{i:k})(m_{i:j} - m_{i:j-1})(\omega_M - m_{i:j})$, and $(m_{j+1:k} - m_{i:j})(m_{i:j} - m_{i:j-1})(\omega_j - \omega_M)$, respectively, yields (7).

Intuitively, in either case ($m_{i:k} \le \omega_M$ or $m_{i:k} \ge \omega_M$), we have four linear inequalities ((6), (9), (10), and (11) or (12)) with 6 variables ($V(m_{i:j-1})$, $V(m_{i:j})$, $V(\omega_M)$, $V(\omega_j)$, $V(m_{j+1:k})$, and $V(m_{i:k})$). Fourier-Motzkin elimination of three variables ($V(m_{i:j-1})$, $V(\omega_M)$, and $V(\omega_j)$) yields one inequality (7) with 3 variables ($V(m_{i:j})$, $V(m_{j+1:k})$, and $V(m_{i:k})$). Figure A.1 illustrates why (7) holds. *Q.E.D.*

We now show that the optimal deterministic upper censorship cutoff coincides with the optimal stochastic upper censorship cutoff. For each $z \in [\omega_1, \omega_n]$, define

$$j(z) = \max\{i \in \{1, \dots, n\} : \omega_i \le z\},$$

$$q(z) = \frac{z - \omega_{j(z)}}{\omega_{j(z)+1} - \omega_{j(z)}},$$

$$m(z) = \frac{(1 - q(z)) f_{j(z)} \omega_{j(z)} + \sum_{i > j(z)} f_i \omega_i}{(1 - q(z)) f_{j(z)} + \sum_{i > j(z)} f_i},$$

$$W(z) = \sum_{i < j(z)} f_i V(\omega_i) + q(z) f_{j(z)} V(\omega_{j(z)}) + \left((1 - q(z)) f_{j(z)} + \sum_{i > j(z)} f_i \right) V(m(z)).$$

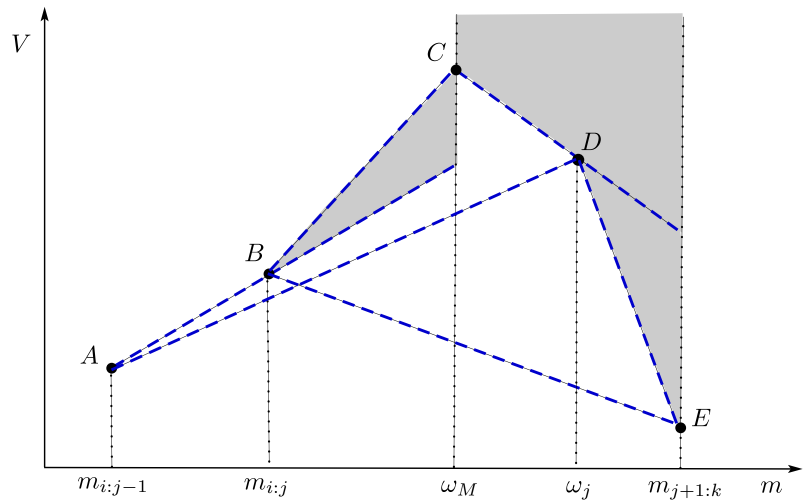

FIGURE A.1.—Upper censorship when V is s-shaped

Note: Point B is above line AD, because $W \geq W_+$. Point C is above line AB, because V is convex on $[0, \omega_M]$. Point E is below line CD, because V is concave on $[\omega_M, 1]$. Point $(m_{i:k}, V(m_{i:k}))$ is in the shaded area, because V is convex on $[0, \omega_M]$ and concave on $[\omega_M, 1]$. Condition (7) states that the shaded area is above line BE.

Thus, every $z \in [\omega_1, \omega_n]$ represents a stochastic upper censorship signal with $(\omega_{j(z)}, q(z))$, where $m(z)$ is the expected state conditional on the pooling signal realization and $W(z)$ is the value of this signal. Conversely, every stochastic upper censorship signal with (ω_j, q) in $\{\omega_1, ..., \omega_{n-1}\} \times [0,1]$ can be represented by $z = (1-q)\omega_j + q\omega_{j+1} \in [\omega_1, \omega_n]$.[14] Also note that z represents deterministic upper censorship iff $z \in \{\omega_1, ..., \omega_n\}$.

Observe that $m(z)$ and $W(z)$ are continuous by construction. Letting

$$\Delta(\omega, m) = V(\omega) - V(m) - V'(m)(\omega - m)$$

and taking the derivative of $W(z)$ at $z \notin \{\omega_1, \ldots, \omega_n\}$, we obtain

$$m'(z) = \frac{q'(z) \sum_{i>j(z)} f_i(\omega_i - \omega_{j(z)})}{\left((1-q(z))f_{j(z)} + \sum_{i>j(z)} f_i\right)^2} = \frac{q'(z) f_{j(z)}(m(z) - \omega_{j(z)})}{(1-q(z))f_{j(z)} + \sum_{i>j(z)} f_i},$$

$$W'(z) = q'(z) f_{j(z)}(V(\omega_{j(z)}) - V(m(z)) + m'(z)\Big((1-q(z))f_{j(z)} + \sum_{i>j(z)} f_i\Big)V'(m(z))$$

[14] An upper censorship with (ω_n, q), for any $q \in [0,1]$, is the same as the upper censorship with $(\omega_{n-1}, 1)$.

$$= q'(z) f_{j(z)} \big(V(\omega_{j(z)}) - V(m(z)) - V'(m(z))(\omega_{j(z)} - m(z)) \big)$$
$$= \frac{f_{j(z)}}{\omega_{j(z)+1} - \omega_{j(z)}} \Delta(\omega_{j(z)}, m(z)).$$

CLAIM 1: *For all* $\omega_1 \leq z < z' < \omega_n$*, we have*

$$\Delta(\omega_{j(z)}, m(z)) \leq 0 \implies \Delta(\omega_{j(z')}, m(z')) < 0.$$

PROOF: Suppose by contradiction that $\omega_1 \leq z < z' < \omega_n$, $\Delta(\omega_{j(z)}, m(z)) \leq 0$, and $\Delta(\omega_{j(z')}, m(z')) \geq 0$. By definition, $\omega_{j(z)}$ is increasing in z, $m(z)$ is strictly increasing in z, and $\omega(z) < m(z)$, for $z \in [\omega_1, \omega_n)$. Thus, letting $\omega = \omega_{j(z)}$, $\omega' = \omega_{j(z')}$, $m = m(z)$, and $m' = m(z')$, we have $\omega \leq \omega'$, $m < m'$, $\omega < m$, and $\omega' < m'$. By integration by parts,

$$\int_{\omega}^{m} V''(x)(x-\omega)\mathrm{d}x = V'(x)(x-\omega)\big|_{\omega}^{m} - \int_{\omega}^{m} V'(x)\mathrm{d}x$$
$$= V'(m)(m-\omega) - (V(m) - V(\omega)) = \Delta(\omega, m).$$

Since V is strictly convex on $[0, \omega_M]$ and strictly concave on $[\omega_M, 1]$, we have $V''(x) > 0$ for almost all $x \in [0, \omega_M]$ and $V''(x) < 0$ for almost all $x \in [\omega_M, 1]$. So, since $\omega < m$ and $\Delta(\omega, m) \leq 0$, we have $m > \omega_M$. Similarly, since $\omega' < m'$ and $\Delta(\omega', m') \geq 0$, we have $\omega' < \omega_M$. Then we obtain a contradiction

$$\Delta(\omega', m') = \int_{\omega'}^{m'} V''(x)(x-\omega')\mathrm{d}x < \int_{\omega'}^{m} V''(x)(x-\omega')\mathrm{d}x$$
$$= \int_{\omega'}^{m} V''(x)(x-\omega)\frac{x-\omega'}{x-\omega}\mathrm{d}x \leq \frac{\omega_M - \omega'}{\omega_M - \omega}\int_{\omega'}^{m} V''(x)(x-\omega)\mathrm{d}x$$
$$\leq \frac{\omega_M - \omega'}{\omega_M - \omega}\int_{\omega}^{m} V''(x)(x-\omega)\mathrm{d}x = \frac{\omega_M - \omega'}{\omega_M - \omega}\Delta(\omega, m) \leq 0,$$

where the first inequality holds because $\omega_M < m < m' \leq 1$ and V is strictly concave on $[\omega_M, 1]$, the second inequality holds because V is convex on $[0, \omega_M]$, concave on $[\omega_M, 1]$, and $(x-\omega')/(x-\omega)$ is increasing in x, the third inequality holds because $0 \leq \omega \leq \omega' < \omega_M$ and V is convex on $[0, \omega_M]$, and the fourth inequality holds because $\Delta(\omega, m) \leq 0$. *Q.E.D.*

Since $f_{j(z)}/(\omega_{j(z)+1}-\omega_{j(z)})>0$, Claim 1 implies that $W'(z)$ is strictly single crossing from above on $[\omega_1,\omega_n)$. This implies that the optimal unrestricted signal is unique and is stochastic upper censorship with some cutoff ω^*. Furthermore, this implies that each optimal monotone signal is deterministic upper censorship with the same cutoff ω^* and some $q^{**}\in\{0,1\}$. *Q.E.D.*

A.2. *Proof of Theorem 2*

It is convenient to represent a monotone signal by a *pooling set* $P\subset[0,1]$ of states that are not separated by this signal. Since the state is continuous, w.l.o.g., each pooling interval is open. Thus, the pooling set is a union of some disjoint nonempty open intervals, $P=\bigcup_i(\underline{\omega}_i,\overline{\omega}_i)$.[15] A pooling set P corresponds to the monotone signal μ_P given by

$$\mu_P(\omega)=\begin{cases}\omega, & \omega\notin(\underline{\omega}_i,\overline{\omega}_i)\text{ for all }i,\\ m_i, & \omega\in(\underline{\omega}_i,\overline{\omega}_i)\text{ for some }i,\end{cases}$$

where $m_i=\mathbb{E}[\omega|\omega\in[\underline{\omega}_i,\overline{\omega}_i]]$. The distribution G_P of $\mu_P(\omega)$ is given by

$$G_P(\omega)=\begin{cases}F(\omega), & \text{if }\omega\notin(\underline{\omega}_i,\overline{\omega}_i)\text{ for all }i,\\ F(\underline{\omega}_i), & \text{if }\omega\in(\underline{\omega}_i,m_i)\text{ for some }i,\\ F(\overline{\omega}_i), & \text{if }\omega\in[m_i,\overline{\omega}_i)\text{ for some }i.\end{cases}$$

Solving the monotone persuasion problem is thus equivalent to finding an *optimal pooling set* P that maximizes $\int_0^1 V(\omega)\mathrm{d}G_P(\omega)$.

LEMMA 2: *Each optimal pooling set P takes one of the following forms:*

1. *Interval disclosure $P=[0,\omega_L^*)\cup(\omega_R^*,1]$ with $m_L^*<\omega_L<\omega_L^*<\omega_R^*<\omega_R<m_R^*$.*
2. *Cutoff rule $P=[0,\omega^*)\cup(\omega^*,1]$ with $m_L^*<\omega_L<\omega_R<m_R^*$.*
3. *No disclosure $P=[0,1]$.*

PROOF: We start with two simple claims.

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[15] We define open sets in $[0,1]$ rather than in $\mathbb{R}$; e.g., $[0,1/2)\cup(1/2,1]$ is open.

CLAIM 2—Kolotilin et al. (2022): *Let P be an optimal pooling set.*

1. *If V is strictly concave on $[0,1]$, then $P=[0,1]$.*
2. *If V is s-shaped on $[0,1]$, then $P=(\omega^*,1]$, with $\omega^*<\omega_M<\mathbb{E}[\omega|\omega\in[\omega^*,1]]$.*

PROOF: Parts 1 and 2 follow from Corollary 1 and Theorem 1 in Kolotilin et al. (2022). *Q.E.D.*

CLAIM 3: *Each optimal pooling set P has the following properties.*

1. *Each separating interval $[\overline{\omega}_i,\underline{\omega}_{i+1}]$, with $\overline{\omega}_i<\underline{\omega}_{i+1}$, is such that $[\overline{\omega}_i,\underline{\omega}_{i+1}]\subset[\omega_L,\omega_R]$.*
2. *There is at most one pooling interval $(\underline{\omega}_i,\overline{\omega}_i)$, with $\underline{\omega}_i<\overline{\omega}_i$, such that $m_i\in[0,\omega_L]$.*

PROOF: To prove Part 1, suppose by contradiction that either $\overline{\omega}_i<\omega_L$ or $\underline{\omega}_{i+1}>\omega_R$. Then a pooling set that differs from P only in that it pools all states in $(\overline{\omega}_i,\omega_L)$ or $(\omega_R,\underline{\omega}_{i+1})$ yields a strictly higher value by Part 1 of Claim 2, as V is strictly concave on $[0,\omega_L]$ and $[\omega_R,1]$.

To prove Part 2, suppose by contradiction that there are two pooling intervals $(\underline{\omega}_i,\overline{\omega}_i)$ and $(\underline{\omega}_j,\overline{\omega}_j)$, with $\overline{\omega}_i\le\underline{\omega}_j$, such that $m_i,m_j\in[0,\omega_L]$. Then a pooling set that differs from P only in that it pools all states in $(\underline{\omega}_i,\overline{\omega}_j)$ yields a strictly higher value by Part 1 of Claim 2, as V is strictly concave on $[0,\omega_L]$ and the support of G_P conditional on $(\underline{\omega}_i,\overline{\omega}_j)$ is a subset of $[0,\omega_L]$. *Q.E.D.*

Suppose by contradiction that an optimal P does not take Form 1, 2, or 3. If there is a separating interval $[\overline{\omega}_i,\underline{\omega}_{i+1}]$, with $\overline{\omega}_i<\underline{\omega}_{i+1}$, then $P=[0,\overline{\omega}_i)\cup(\underline{\omega}_{i+1},1]$, which is Form 1, leading to a contradiction. Indeed, by Part 1 of Claim 3, we have $[\overline{\omega}_i,\underline{\omega}_{i+1}]\subset[\omega_L,\omega_R]$. By Part 2 of Claim 2, we have $P\cap[\overline{\omega}_i,1]=(\underline{\omega}_{i+1},1]$, as V is s-shaped on $[\overline{\omega}_i,1]\subset[\omega_L,1]$. Analogously, $P\cap[0,\underline{\omega}_{i+1}]=[0,\overline{\omega}_i)$.

Next, suppose that there is no separating interval. Since P does not take Forms 2 or 3, P has two pooling intervals, $(\underline{\omega}_i,\overline{\omega}_i)$ and $(\underline{\omega}_{i+1},\overline{\omega}_{i+1})$, with $\overline{\omega}_i=\underline{\omega}_{i+1}$, and $m_i\ge\omega_L$ or $m_{i+1}\le\omega_R$. W.l.o.g., suppose $m_{i+1}\le\omega_R$. By Part 2 of Claim 3, $m_{i+1}>\omega_L$. By Part 2 of Claim 2, $\underline{\omega}_{i+1}<\omega_L$. In summary, we have

$$m_i<\overline{\omega}_i=\underline{\omega}_{i+1}<\omega_L<m_{i+1}\le\omega_R.$$

Let $\omega = \overline{\omega}_i = \underline{\omega}_{i+1}$. Since P is optimal, the marginal effect of changing ω should be 0, so, letting

$$\hat{V}(x) = V(m_i)(F(x) - F(\underline{\omega}_i)) + V(m_{i+1})(F(\overline{\omega}_{i+1}) - F(x)), \quad \text{and}$$
$$\delta(x) = V(m_i) + V'(m_i)(x - m_i) - V(m_{i+1}) - V'(m_{i+1})(x - m_{i+1}),$$

we have

$$\hat{V}'(\omega) = V'(m_i)\frac{\mathrm{d}m_i}{\mathrm{d}\omega}(F(\omega) - F(\underline{\omega}_i)) + V'(m_{i+1})\frac{\mathrm{d}m_{i+1}}{\mathrm{d}\omega}(F(\overline{\omega}_{i+1}) - F(\omega))$$
$$+(V(m_i) - V(m_{i+1}))f(\omega)$$
$$= [V(m_i) + V'(m_i)(\omega - m_i) - V(m_{i+1}) - V'(m_{i+1})(\omega - m_{i+1})]f(\omega) = \delta(\omega)f(\omega) = 0.$$

Thus, $\hat{V}'(\omega) = 0$ iff $\delta(\omega) = 0$. Since V is strictly concave on $[m_i, \omega_L] \subset [0, \omega_L]$ and strictly convex on $[\omega_L, m_{i+1}] \subset [\omega_L, \omega_R]$, we have $V(m_i) + V'(m_i)(\omega_L - m_i) > V(\omega_L)$ and $V(m_{i+1}) + V'(m_{i+1})(\omega_L - m_{i+1}) < V(\omega_L)$, respectively. Thus,

$$\delta(\omega_L) = V(m_i) + V'(m_i)(\omega_L - m_i) - V(m_{i+1}) - V'(m_{i+1})(\omega_L - m_{i+1}) > 0.$$

Next, since $\delta(x)$ is linear in x and $m_i < \omega < \omega_L$, we have

$$\delta(m_i) < \delta(\omega) = 0 < \delta(\omega_L). \tag{13}$$

Since $\delta(m_i) = (m_{i+1} - m_i)V'(m_{i+1}) + V(m_i) - V(m_{i+1})$, by (13), we have

$$V'(m_{i+1}) < \frac{V(m_{i+1}) - V(m_i)}{m_{i+1} - m_i}. \tag{14}$$

Let

$$\nu(x) = V(x) - \frac{m_{i+1} - x}{m_{i+1} - m_i}V(m_i) - \frac{x - m_i}{m_{i+1} - m_i}V(m_{i+1}).$$

We have

$$\nu'(x) = V'(x) - \frac{V(m_{i+1}) - V(m_i)}{m_{i+1} - m_i}.$$

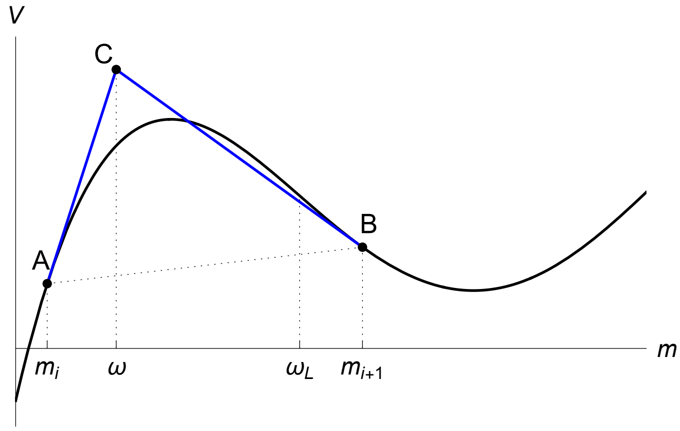

FIGURE A.2.—Interval disclosure when V is m-shaped.

Note: The tangents to V at points A and B intersect at point C, because $\delta(\omega) = 0$. Condition $\nu(x) > 0$ for $x \in (m_i, m_{i+1})$ states that V is above line segment AB.

Since V is strictly concave on $[m_i, \omega_L] \subset [0, \omega_L]$ and strictly convex on $[\omega_L, m_{i+1}] \subset [\omega_L, \omega_R]$, $V'(x)$ is strictly quasiconvex on $[m_i, m_{i+1}]$, and so is $\nu'(x)$. Moreover, by (14), $\nu'(m_{i+1}) < 0$. Hence, $\nu'(x)$ is strictly single crossing from above on $[m_i, m_{i+1}]$. Thus, $\nu(x)$ is strictly quasiconcave on $[m_i, m_{i+1}]$. By $\nu(m_i) = \nu(m_{i+1}) = 0$, it follows that $\nu(x) > 0$ for all $x \in (m_i, m_{i+1})$. Figure A.2 illustrates why $\nu(x) > 0$. Hence, a pooling set that differs from P only in that it pools all states in $(\underline{\omega}_i, \overline{\omega}_{i+1})$ yields a strictly higher value, leading to a contradiction. *Q.E.D.*

By Proposition 3 in Kolotilin (2018), $P = [0, \omega_L^*) \cup (\omega_R^*, 1]$, with $\omega_L < \omega_L^* < \omega_R^* < \omega_R$, is optimal iff (2) and (3) hold. Moreover, by Lemma 2, $m_L^* < \omega_L$ and $m_R^* > \omega_R$. So, Part 1 of Theorem 2 follows. If such $\omega_L^* < \omega_R^*$ do not exist, then P takes Form 2 or 3 of Lemma 2. Clearly, $P = [0, 1]$ is suboptimal iff (5) holds for some $\omega^* \in (0, 1)$. Moreover, if $P = [0, \omega^*) \cup (\omega^*, 1]$ is optimal, then (4) holds, and $m_L^* < \omega_L$ and $m_R^* > \omega_R$ by Lemma 2. So, Parts 2 and 3 of Theorem 2 follow. *Q.E.D.*

A.3. *Proof of Proposition 1*

Let $X \subset C$. For each $\omega \in [0, 1]$, define

$$\underline{c}_X(\omega) = \sup\left(\{c \in C \backslash X : c \leq \omega\} \cup \{0\}\right), \quad \overline{c}_X(\omega) = \inf\left(\{c \in C \backslash X : c > \omega\} \cup \{1\}\right),$$

$$\text{and} \quad \mu(\omega) = \mathbb{E}[\theta | \theta \in [\underline{c}_X(\omega), \overline{c}_X(\omega)]].$$

Observe that μ is a monotone signal such that $G_\mu = G_X$. Thus, $\mathcal{G}_C \subset \mathcal{G}_M$.

Conversely, let μ be a monotone signal. For each m such that $\mu(\omega) = m$ for some $\omega \in \operatorname{supp}(F)$, define

$$\underline{x}_\mu(m) = \inf\{\omega \in \operatorname{supp}(F) : \mu(\omega) = m\}, \quad \overline{x}_\mu(m) = \sup\{\omega \in \operatorname{supp}(F) : \mu(\omega) = m\},$$
$$\text{and} \quad X = \left(\bigcup\nolimits_{m \in \mu(\operatorname{supp}(F))} (\underline{x}_\mu(m), \overline{x}_\mu(m)]\right) \bigcap C.$$

Observe that X is a censorship policy such that $G_X = G_\mu$. Thus, $\mathcal{G}_M \subset \mathcal{G}_C$. *Q.E.D.*